Edexcel GCSE

Coursework Guide – Using and Applying Mathematics

Edexcel GCSE in Mathematics (1387/1388)

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Introduction

For the first time, as a component of GCSE assessment, all students will be required to produce a minimum of **one** coursework task which demonstrates their ability to 'Use and Apply Mathematics'. This task will carry a weighting of 10% of the final GCSE mark and will be a part of the overall 20% available to the candidate's assessment in 'Using and Applying Mathematics' (AO1). The task submitted must be from the sections of the National Curriculum relating to 'Shape and Space' (AO3) or 'Number and Algebra' (AO2).

Candidates will need to have good investigational skills and they will need to be trained in using and applying their mathematical knowledge in a variety of circumstances.

The aim of this booklet is to clarify the requirements for a coursework task by looking at the requirements for individual marks of the assessment procedure and to support this with examples of candidates work. The exemplar material will be used to show the minimum requirement for an individual mark.

This Coursework guidance should be used in conjunction with the General Criteria and the Elaboration Document issued by QCA.

EDEXCEL PHILIOSOPHY AND PRINCIPLES

— The coursework component is more than merely a method of assessment, it is, perhaps above all else, an instrument to facilitate and encourage curriculum development in schools and colleges. Few would doubt the need to consider such developments in mathematics, particularly in relation to the recommendations on Methodology set out in the Cockcroft report and towards developing mathematics at Key Stage 4 of the National Curriculum.

— Coursework should encourage good practice. The elements of such, as defined in the Cockcroft report (paragraph 243) and the Non-Statutory Guidance for the National Curriculum, should be in evidence whilst students are undertaking coursework activities.

— Coursework should be an integral part of the mathematics curriculum and not simply a bolt-on exercise aimed at satisfying new assessment criteria.

— The teacher should play an active role by discussing with students what they are doing and why. In this way the student can be encouraged to talk about mathematics and rationalise why they have developed the task along particular lines. Help given need only be recorded when it has been necessary to redirect a student in order to achieve an assessable outcome.

The nature of the coursework tasks

Coursework tasks covering the two areas of ‘Number and Algebra’ (AO2) and ‘Shape and Space’ (AO3) could be considered to fall into the following types.

1. ALGEBRAIC INVESTIGATIONS

These tasks are normally initiated in a numerical context but should enable the candidates to progress and find algebraic generalisations. The generalisations discovered will obviously depend upon the ability of the individual candidates as will the various techniques used in the work.

2. MINIMISING AND MAXIMISING INVESTIGATIONS

These tasks are often set in the context of ‘Shape and Space’. The candidates have to work through the task in a systematic way in an attempt to find the minimum/maximum values for area or volume. These tasks often involve graphical work and a clear understanding about the justification for maximum/minimum values of a variety of different shapes.

3. TASKS INVOLVING BREAK-EVEN POINTS

These tasks are often set up in real-life situations where two or more payment schemes are considered to see which one is the best option. The work involves looking for ‘break-even’ points, normally using a graphical, algebraic or trial and improvement approach. To progress these tasks it is essential that the candidates can develop more general algebraic equations and examine them analytically.

4. PERMUTATION AND COMBINATION TASKS

These tasks will need the candidates to have, as a primary requirement, good listing skills. They will also need to be strategic in their approach towards the work and eventually moving the task into a more general solution. This could involve the factorial notation or algebraic expressions.

5. FIBONACCI TASKS

These tasks involve systematic listing and careful checking is essential to ensure that no possibilities have been omitted. The sequences, once recognised, would need to be carefully tested for validity. The task would then be progressed, through a consideration of the overall generalisations, using a variety of different approaches.

Key issues for delivery

To ensure that the candidates can achieve their maximum potential in a coursework task it is essential that they have the necessary basic skills. These skills should have been established over a period of time. During the lower years in the school situation the students should have had access to coursework tasks suitable to their ability. This enables them to develop the skills for success in the GCSE situation.

ESSENTIAL SKILLS

1. SYSTEMATIC APPROACH

The vast majority of the tasks require the students to be systematic in their approach if they are to achieve any success. This could involve listing outcomes in order, drawing diagrams in sequence or moving a shape around on a number grid. Any tasks where listings are appropriate the candidate has to have a good system that will ensure that they have all of the possibilities, and no duplicates. The skill of listing requires continual practise and candidates should have had considerable opportunity to demonstrate this technique.

2. COMMUNICATION

One of the important aspects of coursework is the presentation of the results. The candidates will need to know, therefore, how to present numerical information in a table, making sure that there is sufficient relevant information available. A good table of results will enable patterns and hence generalisations to be spotted quicker.

Graphical communication is also very important and candidates should have this skill in their armoury. The use of the correct type of graphs for the task under consideration should be encouraged and again, will need repeated practise throughout the school curriculum.

It is also very important that the candidates communicate to the reader the reasons ‘ why ‘ they are using a certain means of presentation. A comment, such as, ‘If I put my results into a table then I may be able to spot a pattern’ quite clearly shows what is happening and the reader can follow the work.

3. LOOKING FOR GENERALISATIONS

To progress the majority of tasks, a generalisation of one kind or another needs to be found. Often the generalisation is given verbally initially, and then algebraically. It is essential, therefore, that the candidates have appropriate skills to help them find these generalisations. Expressing generalisations in linear and quadratic form are part of the specifications that are examined on the written paper. It would seem appropriate, therefore, that mini investigations could form part of the teaching when this aspect of the work is under consideration. Students would then be aware of the procedure when meeting a similar idea in a coursework investigation.

Far too often candidates think that the ‘differencing technique’ is essential in every task. Candidates need to be shown when it is appropriate and the importance of the initial information being consecutive in nature.

4. STRUCTURE OF THE TASK

Most of the tasks, where an algebraic generalisation is involved, can be investigated through their structure. This aspect of coursework is one to be encouraged because it has many advantages to the tried system of differencing/pattern spotting.

- Gives a better understanding of the task
- Enables generalisations to be obtained more quickly
- Often saves the candidates time
- Enables better access to the higher marks
- Candidates enjoy this approach.

There is a need, therefore, to build this aspect of the work into the teaching situation from a very early age. The question 'why is something happening?' as opposed to 'what' is to be encouraged.

5. TESTING

Where appropriate, the candidates should be regularly checking to see if their results are correct and sensible. This aspect is particularly important in strand 3 assessment of coursework and one that should be encouraged.

6. EXTENDING THE TASK

In order to achieve the higher marks in coursework the candidates should always be looking at ways in which the task could be extended. The candidates need to be asking questions along the lines of:

- What can I change/alter in the task?
- How could I achieve these new changes?
- Which new mathematical techniques could I employ?
- Can I introduce another feature into the task?

7. EXPLAINING

Throughout the work the candidates should be explaining what they have done. They should be trying to show how they obtained their answers without the constant comment 'I spotted'. The necessary techniques need to be taught to the candidates, therefore, to enable them to justify the results that they have obtained. At the top end of the mark range this can often be seen through the algebraic techniques/approaches that they have used but lower down this is not so obvious.

Classroom practice

Several major issues have arisen which cause concern to teachers:

Introduction of a Coursework Task

Edexcel will provide teaching notes and partial solutions for all tasks produced for 2003 and beyond.

A coursework task is essentially very different from a written examination. Therefore the tasks should **NOT** be introduced without any form of explanation. The task is a written communication between the setter and the candidate – but it is not expected that the tasks will be issued without introduction or explanation. However, the introduction, explanation and examples given should not form part of the assessment.

Teachers may find it helpful to use interactive class or group sessions as a general introduction to each topic. This is not to say, of course, that candidates should be given precise instructions on how to tackle the task, although unfamiliar pieces of vocabulary can be explained.

Details of courses run by Edexcel on methodology and approaches appropriate to coursework can be obtained from the INSET Office.

Time

There is no time limit laid down for the completion of a coursework task. It is left to the centres, therefore, to decide how much time they are going to allow a candidate to devote to a task.

Candidates should not be discouraged from working but time should be spent effectively. For example, colouring in all the regions produced by the diagonals of a regular twenty-sided polygon or drawing a diagram to show all the 64 moves in the Towers of Hanoi for 7 discs, are not really mathematical activities, gain candidates little credit and are not therefore effective uses of time.

Equally, candidates should be encouraged to generalise as quickly as possible, but always with sufficient supporting evidence, rather than produce pages of repetitious work that will gain no extra credit. This is particularly so in the case of more able candidates expected to achieve the higher grades. Teachers can encourage candidates to generalise by appropriate questioning without penalty to the student.

The question of time is also relevant to the teacher marking the tasks. Most teachers have tackled this in a very professional manner and it is hoped that as they continue to gain experience of assessing coursework, the amount of time taken will be reduced. The production of a centre's own assessment guidance not only serves to help internal moderation but also helps to reduce the amount of time spent marking.

How much help can a teacher give?

Good practice should prevail. It is not good practice to allow candidates to flounder, nor is it good practice to show them solutions to problems. Candidates can be asked questions of a type given on pages 7 and 8 without penalty. However, where a candidate cannot progress to an assessable outcome the minimum help necessary to enable him/her to achieve this should be given and recorded on the Student's Task Form. This help would need to be taken into account when determining a candidate's final mark in each strand.

As an aid to explaining what we mean by undue help, that is help which was necessary for a candidate to progress to an assessable outcome, we offer a simple illustration.

Suppose an investigation led to a set of results as:

$1 \rightarrow 3$
$2 \rightarrow 5$
$3 \rightarrow 7$
$4 \rightarrow 9$
$5 \rightarrow 11$ etc

Generalising this as:

$$n \rightarrow 2n + 1$$

is an **assessable outcome.**

A candidate might make a mistake in collecting the data and obtain results.

$1 \rightarrow 3$
$2 \rightarrow 5$
$3 \rightarrow 7$
$4 \rightarrow 9$
$5 \rightarrow 11$ etc

On this data the result cannot be easily generalised – and certainly cannot be $n \rightarrow 2n + 1$.

Clearly the teacher **should not** leave the candidate to flounder. It would be good practice for teachers to make comments such as:

Have another look at your results, or
One of these does not fit the pattern, or
Are you sure about this one?

We would not expect such help to be recorded or taken into account when the work is being assessed.

However, if the only way the candidate can reach the assessable outcome of $n \rightarrow 2n + 1$ is by the teacher taking action such as:

"This result ($4 \rightarrow 8$) is wrong and should be nine," then this action should be recorded.

There is no formula defining how grades or marks should be adjusted in accordance with the help given, professional judgement should be applied.

How can we be certain that this is the student's own work?

Unless a piece of work is carried out under timed examination conditions you cannot guarantee that it is a candidate's own unaided work. With work carried out over a period of time candidates will discuss amongst themselves, talk to parents and others, read books and journals; this is to be encouraged.

Since sole ownership of a piece of work cannot be guaranteed, teachers are asked to check that a candidate understands what he/she has submitted in response to a task and that any answer, algorithm, generalisation or conclusion can be derived from the work that precedes it.

Teachers must undertake sufficient direct supervision of coursework for a candidate's work to be authenticated with confidence.

The following areas are offered as a guide to help teachers structure their classroom assessment. The questions shown within each area are descriptive as actual questions asked could be directly relevant to the activity being assessed. The list provides a source of possible questions.

Questions should generally be open-ended so as to allow the candidate the greatest opportunity to show what has been learnt. Closed questions may be required to obtain clarification of answers given. Teachers are warned of the danger of asking leading questions, i.e. questions likely to bias an answer. Throughout the process it is essential that the candidate remains in control of the investigation.

(*a*) **Assumptions, precautions, starting points**

(i) Were there any things you had to find out before a task could be started?
(ii) What things did you assume during your work? (Rooms being rectangular, lines being parallel, etc.)
(iii) What sort of accuracy did you think would be needed? How did you decide this? Were there any precautions you had to make? (Zeroing scales before use, taking measurements more than once, questionnaire not biased, etc.)

(*b*) **Methods of recording and reporting**

(i) How did you record your information?
(ii) Why did you choose this method?
(iii) Did you consider any other methods?
(iv) What information do you want your results to convey?
(v) Did you think of any other ways of doing this?
(vi) Is there anything wrong with … (such and such a method)?

(*c*) **Results**

(i) How did you reach your results?
(ii) What do they mean?
(iii) Were they the ones you expected?
(iv) Were there any results which did not match the rest? What did you do about this?

(*d*) **Checking**

(i) Tell me how you checked your work.
(ii) Were there any things you had to correct?
(iii) Were there any things you feel you ought to have checked?

(*e*) **False leads**

(i) Were there any parts of your work which you started and then had to scrap and start again? Why?
(ii) Did this help you to choose a new approach?
(iii) Were there any ways you could have continued with your original approach?

(*f*) **Extension**

(i) Having started with this task, can you think of any ways you could develop it and do some further work?
(ii) What would be your new objectives? How would you set about achieving them?

(iii) Has this work given you any other ideas which you would like to explore?

(*g*) **What if…**

Suppose (something) had been (something else), what effect would this have had on your work?

Marking

Teachers are encouraged to mark candidates work in the normal way so that they may receive adequate feedback. Correction of mathematical errors and comments also assist the Moderator when standardising a centre's marks.

Teachers must also annotate each piece of coursework to show how the mark for each strand has been awarded in relation to the assessment guidance and general assessment criteria for AO1. E.g. as a minimum 7(1), 6(2), 5(3), where evidence of this achievement is shown in the coursework.

Standardising

Standardisation is a two way process: **Internal Standardisation** by the teachers at a centre followed by **External Standardisation** carried out between centres by an Edexcel appointed Moderator.

Internal Standardising

Edexcel will provide INSET, addressing the activities, assessment guidance, general assessment criteria for AO1 and arrangements for internal moderation. Centres are advised to attend local support meetings.

Before introducing a task to students it is important that all teachers involved should work through it, meet together to discuss expected outcomes and produce their own assessment guidance, particularly if these differ from those provided by Edexcel.

Upon completion, the tasks should be assessed. Teachers must then meet together to discuss and agree standards before assigning marks (see specification), thus ensuring effective internal standardisation.

External Standardisation

Pre-printed OPTEMS forms will be despatched to centres along with the clearance documents issued after their entries have been processed. The mark out of 24 must be entered on the OPTEMS forms and then bar-coded. The top copy of the OPTEMS is sent to Edexcel and the second copy to the Moderator together with the work of those candidates with an asterisk beside their name on the OPTEMS.

On the basis of the sample seen, the Moderator may need to adjust a centre's marks to bring them in line with the standards set by the Principal Moderator at the Coordination Meeting. Where a problem is identified, the Moderator may request a further sample of the work of the candidates. Normally, the centre's rank order will not be altered. A listing of standardising marks will be produced at about the time the results are issued. A report on the work of the centres will be provided as early in the autumn term as possible. Coursework may be retained by the Foundation for archiving or research purposes.

Ideas for tasks

During the course the candidates should be given tasks that allow them the opportunity to use knowledge, skills and understanding contained in AO2 and AO3 to demonstrate the ability to use and apply mathematics as given in AO1. These tasks could be practical and/or investigational and should involve the use of ICT as appropriate,

Edexcel will provide tasks for centres to select and integrate into their own schemes of work. Centres taking option A may choose to use these tasks, generate their own tasks or use a mixture or centre and Edexcel tasks. Centres taking option B must submit tasks from those set by Edexcel.

Edexcel will publish a separate booklet with these tasks.

Where centres opt to set their own coursework tasks it is not necessary to receive prior approval from Edexcel. However, it is advisable to contact Edexcel to advise on the suitability of the tasks and the correctness of the assessment guidance being used by the centre.

Incorporating the wider curriculum

The students' work on a coursework task could be used to compile evidence for their Key Skills portfolio. Further details of this can be found in the GCSE Mathematics Specifications on pages 89 – 105 (1387) and pages 105 – 119 (1388).

The use of ICT is encouraged within the coursework tasks and many of the tasks set allow for this approach. However, the students should be aware that all of the work that they have done needs to be included and not merely a ' write-up' of the eventual findings.

Throughout the coursework tasks the students will have many opportunities to use ICT. This could be using spreadsheets, creating generalisations, charts and graphical work. Credit in the assessment of the work may be given where the use of ICT has been appropriate.

There will be future publications from Edexcel on the use of ICT and Key Skills in GCSE Mathematics.

Assessment requirements

The general coursework assessment criteria will be used to assess tasks. These criteria are shown in appendix 1 on pages 134—135:

The criteria are sub-divided into three strands. These strands are

Strand 1 – Making and monitoring decisions

Strand 2 – Communicating mathematically

Strand 3 – Developing skills of mathematical reasoning

Candidates should be awarded a mark between 1 and 8 in each of the strands for the work that they have submitted. The marks awarded in each strand should match the statements in the general criteria in the relevant section. Any candidate who fails to satisfy the description for a mark 1 in any of the strands should be awarded a mark of 0 (zero) for that strand.

The marks for each stand are then totalled to give the candidates a final mark out of 24.

Assessing students' work

The assessment of a coursework task is governed by the Assessment Criteria on pages 134–135.

The document 'Elaboration of AO1 Assessment Criteria, has been produced by the Joint Awarding Bodies to help centres with their assessment of the candidates work (see Appendix 2).

It gives an indication as to the minimum requirement for the award of a particular mark.

There are some points that centres need to be aware of when dealing with this document and these have been highlighted below:

Strand 1

- At mark 5 the candidates should be introducing ideas of their own which would enable them to move the task forward. This is normally achieved by changing some feature of the task.
- Mark 6 requires the student to build upon what has already been achieved. They should be using other relevant techniques in the task and extending the task.
- At mark 7 the students should be carrying out some analysis of the situation being investigated. This will normally involve the use of three variables but the task in itself could be complex enough with two variables. It is important that the work being produced is at the appropriate level.
- At mark 8 the students should be working on a 'complex problem' using a range of mathematical techniques from the National Curriculum 'further mathematics' or beyond.

This strand is really about 'doing' the task through a variety of stages, developing the different techniques as the work progresses.

Strand 2

- At mark 4 the students have to be encouraged to use a variety of means of presenting their information. This information must include a commentary to explain what they are doing in terms of the presentation. This commentary should clearly enable the reader to follow the work that has been done. A student may well say' I have put my results into a table to see if a pattern or generalisation can be found.' Another comment could be 'I will draw a graph of the results to see if there is any connection between the number of squares obtained and the number of matches used'
- For mark 5 the student is improving their communication relating to the task. For tasks involving generalisations a move into symbolism would probably be the best way of achieving this. However, the students should be encouraged to say 'why' they are using symbolism rather than just giving an algebraic expression. Again, a student could say 'I have found the generalisation $3n - 6$ which would enable me to work out the solution for any value of n rather than continued drawings.' In the notes above there are suggestions as to the type of words that students could be using. In other types of tasks, the students could achieve this mark by using appropriate diagrams/graphs to enhance their communication. This approach would possibly be more appropriate in the tasks involving AO3.

- Mark 6 is possibly the clearest one to award as it is for consistent use of symbolism at the ‘appropriate’ level. Symbolism must be clearly defined so that there can be no misunderstanding as to the meaning of the symbols used. Any tasks that do not involve the use of algebraic expressions can, obviously, achieve this award by using the appropriate mechanisms at the correct level. The Elaboration Document gives examples of what is meant by symbolism.
- The essence of mark 7 is the student has now presented a ‘convincing’ reasoned argument through the use of language and symbolism, which should be mainly correct. It is very hard to see how a student can be convincing if there are several errors in the work. A general expression derived at this level needs to have the supporting evidence rather than just being stated.
- At mark 8 the student has presented a concise reasoned argument without any errors. The award of this mark has to be supported by all of the appropriate work and at the correct level

This strand is concerned with communicating the work that has been done in the task. There should be an emphasis on accuracy as the marks increase, together with clearly defined symbolism at the higher marks.

Strand 3

This is the one area, which the students find difficult and ample preparatory work needs to have been done before so that the students can comment on their work and explain where the results have come from.

- For the award of mark 3 the students need to make a valid general comment based upon ‘their’ results. The comment must relate to their data but need not be the correct result for the task. For example, the task may require prime numbers to be obtained but the students’ results are 3, 5, 7, 9, 11. A comment from the students that these are all ‘odd numbers’ or ‘they go up in twos’ is sufficient for the award of mark 3.
- Mark 4 requires the students to be looking at any generalisation made in the other strands and then checking to see in this generalisation is actually true. This testing should be carried out using a further case. Testing is an important aspect, as it will help the student to make sure that the results are correct and hence avoid unnecessary work by looking for further results that are not possible.
- Mark 5 is the one that the students find the most difficult as it requires them to look into the structure of the task. They need to relate their results/generalisations to the physical/geometrical/graphical situation of the task. They need to really explain ‘WHY’ the results occur as they do and not ‘I have spotted’.
- The awarding of mark 6 should be for the students who have justified the work done so far through reasoning, logic or proof and can hence move the task forward to the analysis stage. For example, in the Fencing Problem a candidate will have justified the regular cases for the three and four-sided shapes and will comment that they are only going to consider regular polygons in the next stage of the analysis. This mark really is the catalyst for a candidate looking to analyse the situation at mark 7.
- Mark 7 is the ‘bringing together’ of the work, which lead to the overall generalisation in strands one and two. Often the work that the student has done, leading to an overall generalisation as a convincing reasoned argument can be justification enough for the awarding of this mark. However, a mark of 7 in

strand three cannot be awarded without the student having obtained a mark of 7 or 8 in strand 1.

- Mark 8 is awarded for the student justifying /proving the solution that they have obtained to a complex problem. This may be achieved by harmonising two strong arguments to reach the same conclusion.

Using the task specific assessment guidance

The task specific assessment guidance has been designed to aid teacher assessment of AO1 and serve as a guide to help facilitate it.

It is not a mark scheme but a guide as to some likely outcomes candidates may achieve which gives indications of the mark for each strand.

The assessment guidance must always be used in conjunction with the General Criteria and the Elaboration Document when assessing coursework.

Solutions or generalisations to problems must be supported by

(i) Explanations of the methods to find solutions

(ii) Justification

(iii) Logical accounts of work done

(iv) Appropriate forms of recording and presenting their findings

The assessment guidance is not a benchmark and should never be seen as hurdles over which candidates must jump.

Sometimes, when undertaking more open-ended and investigative and problem solving tasks, candidates may proceed along unexpected paths. In these cases, whilst the guidance may act as a guide to assessment, it needs to be applied with the greatest care. In all cases the words 'or mathematical equivalent' must be attached to each indicator at each mark. Reference must always be made to AO1 General Criteria and the Elaboration document produced by QCA. The teacher must be satisfied that the work justifies the marks awarded.

One of the major reasons behind adjustments made through the external moderation process has been that teachers sometimes make a very literal interpretation of the words of the assessment guidance. For instance in the Fencing Problem everything, apart from the 'proof' of the circle result, could be achieved through the use of scale diagrams. However, if candidates use and apply this technique alone, then it is highly unlikely that they could achieve a final mark worthy of anything above a Mark 4 or 5, i.e. in the region of the grade D/C boundary.

In the final analysis, the award for each and every task must be determined by a combination of process and applied content – or the overall **quality** of the mathematics used and applied.

Examples of students' work and moderators' comments

This section includes some examples of students' work relating to the Edexcel tasks. Moderator comments have been included to highlight the type of work necessary for the award of a particular mark.

MARK 1

This section of work is taken from the task 'Opposite Corners'

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3
11	12	13

$1 \times 13 = 13$
$3 \times 11 = 33$

$33 - 13 = 20$
Difference $= 20$

Strand 1 Mark 1

The candidate has shown some understanding by producing a diagram and a result.

Strand 2 Mark 1

A random calculation including a drawing.

Strand 3 Mark 1

Candidate shows an example that shows an understanding of the task.

MARK 2

This section of work is taken from the task 'Opposite Corners'.

I've decided to use the square size of 2 by 3.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

the numbers shaded are 1 and 13, 3 and 11
the product would be $1 \times 13 = 13$
$3 \times 11 = 33$ the difference is 20

the numbers shaded are 45 and 57, 47 and 55
the product would be $45 \times 57 = 2565$
$47 \times 55 = 2585$ the difference is 20

the numbers shaded are 84 and 96, 86 and 44
the product would be $84 \times 96 = 8064$
$86 \times 94 = 8084$ the difference is 20

so I've found out that a rectangle sized 2 by 3 is moved anywhere on a 100 grid, the difference is always <u>20</u>.

Strand 1 Mark 2

The candidate has clearly demonstrated evidence of their own planning by deciding to consider shapes that are 2 by 3 and the correct results are obtained.

Strand 2 Mark 2

The results have been presented in a clear organised way.

Strand 3 Mark 2

The candidate has made simple observation from the data collected.

MARK 3

This section of work is taken from the task ‘T-Totals’

1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	91

T-total	T-shape
37	20
42	21
332	79
147	42
47	22
267	66
62	25

37 – 20 = 17

42 – 21 = 21

47 – 22 = 25

21 – 17 = 4

25 – 21 = 4

The difference of the difference between the T-total and T-shape is always 4 if the T-shape is the one after it

Strand 1 Mark 3

The candidate has started to collect the data necessary to help solve an initial simple task. There is no real system to the work but the checking is evident in that the T-Total increases, as does the T-Shape.

Strand 2 Mark 3

The data is presented in diagrammatic form and in a table of results. There is no commentary to link the forms of presentation so the work does not progress into mark 4.

Strand 3 Mark 3

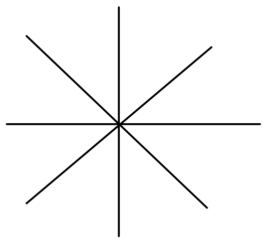
The candidate has made a valid statement based upon the results obtained. There is no attempt to predict any further results or test. The work does not move into mark 4 therefore.

MARK 4

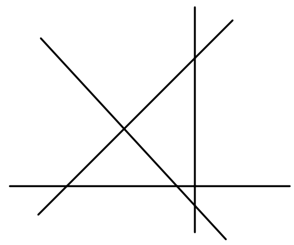
The work in this section is taken from the task ‘Lines, Cross-overs and regions’.

To get the maximum amount of cross-overs, open regions and closed regions you must not cross a line that has already been crossed over, but if there was say 4 lines your 5th line must cross them all.

E.g.



Don’t do this

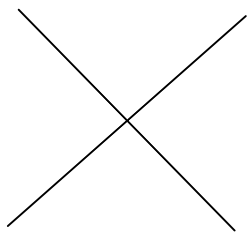


Do this

No. 1

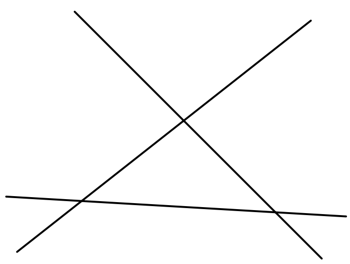
No cross-overs
No closed regions
No open regions

No. 2



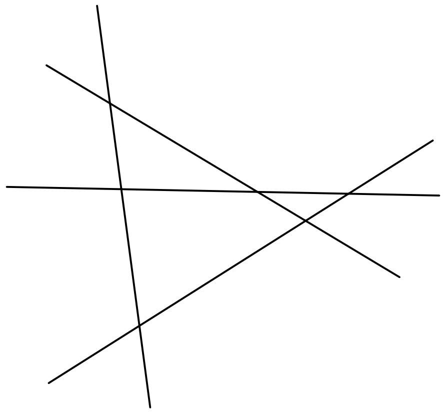
1 cross-over
4 open regions
No closed regions

No. 3



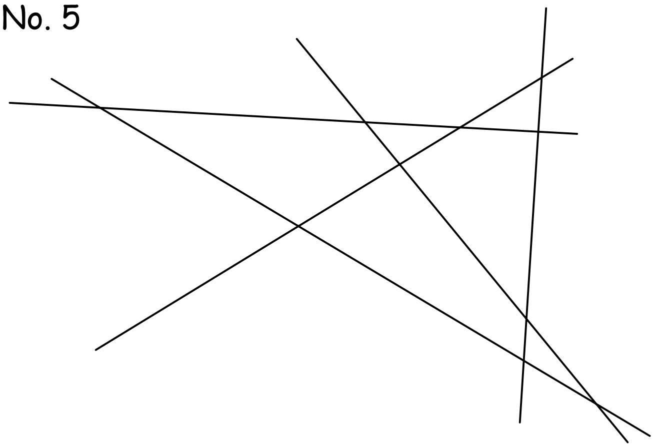
3 cross-overs
6 open regions
1 closed regions

No. 4



6 cross-overs
8 open regions
3 closed regions

No. 5



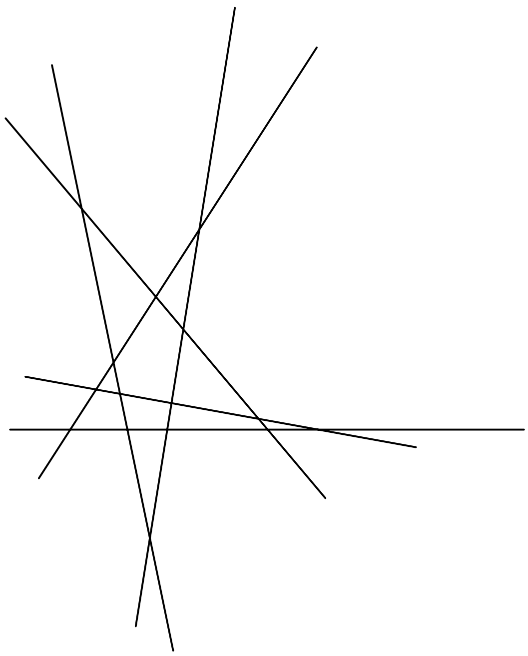
10 cross-overs
10 open regions
6 closed regions

Results Table

	lines	× points	open regions	closed regions	total regions
	1	-	-	-	-
	2	1	4	-	4
	3	3	6	1	7
	4	6	8	3	11
	5	10	10	6	16
prediction →	**6**	**15**	**12**	**10**	**22**

My prediction is correct as you can see from the drawing (drawing No. 6).

No. 6



15 cross-overs
12 open regions
10 closed regions

The first column which shows the cross-over results goes up in a pattern of +2, +3, +4, +5, …

The second column goes up in twos.
The third column goes up in the same pattern as the first column.

The fourth column goes up by +3, +4, +5, +6, …

Now I can extend the table up to ten.

Results Table

lines	× points	open regions	closed regions	total regions
7	21	14	15	29
8	28	16	21	37
9	36	18	28	46
10	45	20	36	56

Strand 1 Mark 4

The original diagram given in the task was 4 lines crossing. The candidate has broken the task down to consider 2/3/4/5 lines crossing each other and has recognised the pattern in the sequence for each area under consideration. These are shown in the table of results.

Strand 3 Mark 4

The candidate has made a prediction based upon the results obtained up to 5 lines crossing and has then predicted the 6 line case and tested it.

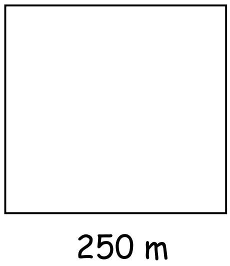
The work is not awarded mark 4 in strand 2, as there is no linking commentary from the diagrams to the table of results.

The work in the following section is taken from the task 'Fencing'.

In this investigation I am trying to find the maximum area for a plot of land with a perimeter (or circumference) of 1000 m. I will be investigating various shapes to try and find which shape will give the largest area within the 1000 m fencing unit. I will find and use different shapes with different side length to find which shape gives the largest plot. I will start with quadrilaterals and then investigate polygons, e.g. hexagons, pentagons, octagons, etc.

I will now start investigating quadrilaterals:

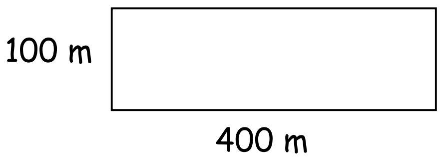
i) Square



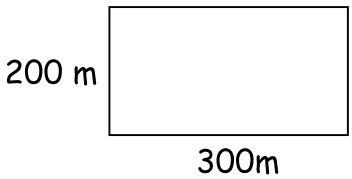
perimeter = $250 \times 4 = 1000$ m
area of square = $250 \times 250 = 62500$ m^2

ii) Rectangle

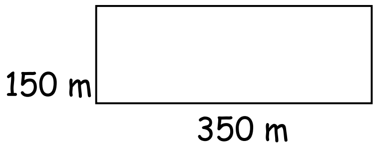
perimeter = $50 \times 2 + 450 \times 2 = 1000$ m
rectangle = $450 \times 50 = 22500$ m^2



perimeter = $400 \times 2 + 100 \times 2 = 1000$ m
area of rectangle = $400 \times 100 = 40000$ m^2



perimeter = $300 \times 2 + 200 \times 2 = 1000$ m
area of rectangle = $300 \times 200 = 60000$ m^2



perimeter = 350 × 2 + 150 × 2 = 1000 m
area of rectangle = 350 × 150 = 52500 m^2

There are many possible variations to the way in which the fencing could be put up but it would take an extremely long time to record every single one. I have drawn 5 different possible solutions to the fencing problem with a difference of 50 m as a range for each step.

I am now going to record my results in a table and try and see any conclusions that I can make.

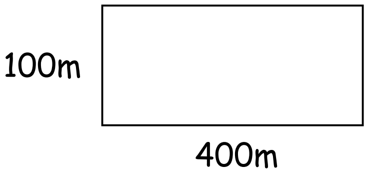
Length (m)	Height (m)	Area (m^2)	Difference in side lengths
250	250	62500	0
200	300	60000	100
150	350	52500	200
100	400	40000	300
50	450	22500	400

From looking at the results in the table I can see that there are far more possibilities than the ones I have drawn for rectangular solutions to the problems and if I attempted to draw them all it would take a very long time. I can also see that if I increase the length and decrease the height, by the range of 50 m each time, it will give the same areas because the height and length are the same but the opposite way round,

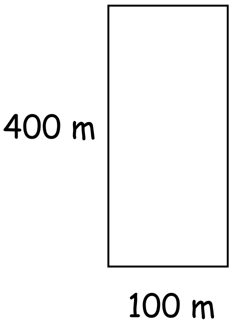
e.g. length = 150 m × 2
height = 350 m × 2 = 1000 m
or height = 150 m × 2
length = 350 m × 2 = 1000 m

I will now draw a graph (not included here) to display all the possibilities of the fencing problem for quadrilaterals which may help me to see a pattern or shape to the results. It will also show me every single solution for quadrilaterals and maybe the relationship between side lengths and areas.

From the graph, I can see that it doesn't show the entire spectrum of solutions so I must now draw two more rectangles around the square shape to show this. The graph doesn't show the entire lot of solutions because it only has the positive solution and not the negative solutions, i.e.



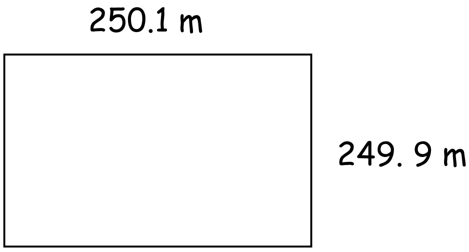
Difference in side lengths
= 400 m – 100 m
= 300 m



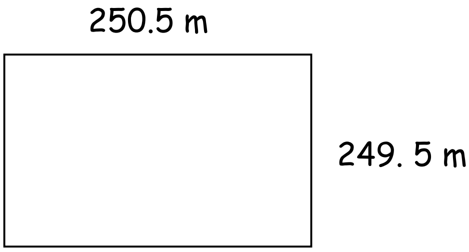
Difference in side lengths
= 100 m – 400 m
= –300 m

As well as the entire range of solutions I must also find out about the maximum areas possible. I have seen that for the rectangles, a square has the largest area.

I will now draw two rectangles with side length measurements either side of the square 250 m $\times$ 250 m to see whether their areas are larger or smaller than that of the square.



Area = $250.1 \times 249.9 = 62499.99\ m^2$



Area = $250.5 \times 249.5 = 62499.75\ m^2$

I will now put all my results in a graph which will show the positive and negative differences in side lengths to produce a symmetrical graph showing clearly that the square is the highest point and so has the largest area.

To make a graph, I will use a spreadsheet:

Width	Length	Area	Difference in side lengths
250	250	62500	0
250.1	249.9	62499.99	0.2
250.5	249.5	62499.75	1
300	200	60000	100
350	150	52500	200
400	100	40000	300
450	50	22500	400
249.9	250.1	62499.99	-0.2
249.5	250.5	62499.75	-1
200	300	60000	-100
150	350	52500	-200
100	400	40000	-300
50	450	22500	-400

I also show the spreadsheet with its formulas.

Strand 2 Mark 4

The candidate has clearly linked together the diagrams and the table of results with a commentary. This commentary indicates to the reader the intentions of the candidate. The reader is able to follow, quite clearly, what is happening in the work.

MARK 5

The work in this section is taken from the task 'Lines, Cross-overs and Regions'.

Lines, Cross-overs and Regions

To investigate the relationships between the number of lines, the maximum number of cross-over points and the maximum number regions.

In this investigation I am going to look at the relationship between the number of lines, the maximum number of cross-over points, and the maximum number of regions by drawing a number of diagrams in which show maximum cross-over points, the number of lines and crossed regions. I will look at the results from my diagrams and try and discover a pattern. Then ultimately find a formula.

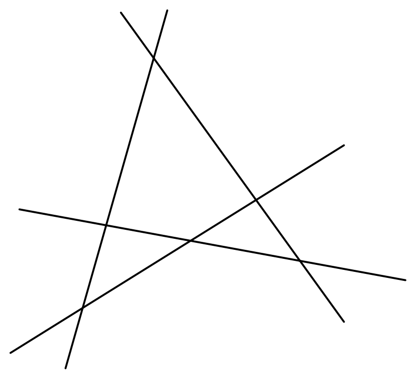
Cross-overs

Here are some diagrams I drew to try and find the maximum number of cross-over points.

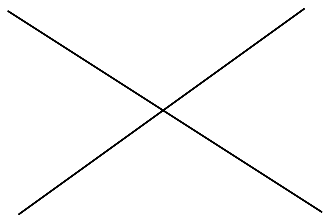
L = No. of lines R = regions CO = Cross- overs

L = 1
CO = 0
R = 2

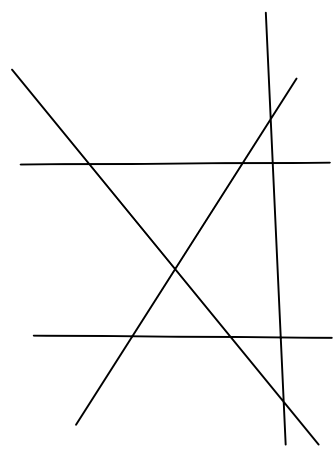
L = 4
CO = 6
R = 11



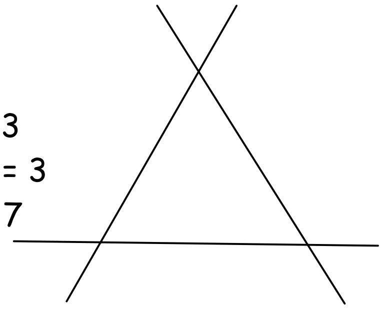
L = 2
CO = 1
R = 4



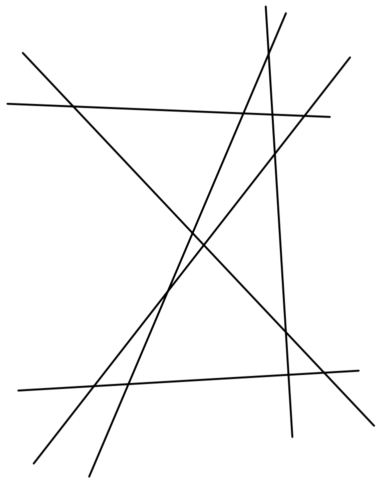
L = 5
CO = 9
R = 15



L = 3
CO = 3
R = 7



L = 6
CO = 14
R = 21



Cross-over Tables

To analyse my results I must set them out into table and graphs.

No of lines	No of cross-over points
1	0
2	1
3	3
4	6
5	9
6	14

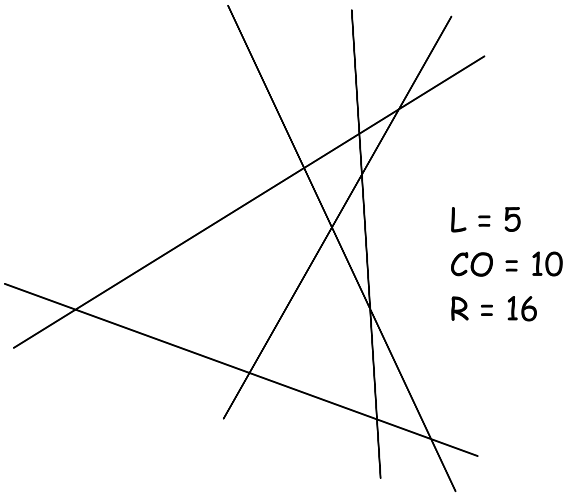
Before I put my results into a graph I looked at them and noticed there was not a constant, clear pattern.

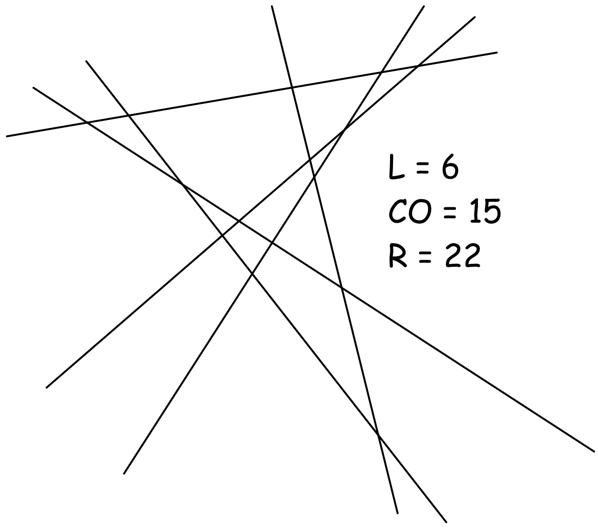
No of lines	No of cross-over points
1	0
	} 1
2	1
	} 2
3	3
	} 3
4	6
	} 3
5	9
	} 5
6	14

As you can see in the first three diagrams (results) there is a clear pattern. In each result the next number has a difference that is one higher than the previous difference. But you can see from where four lines begin to be used the pattern is lost, due to an error in my diagrams. So I must try once again to find the maximum number of cross-overs using 5, then six lines.

Correct diagrams

I have tried once again to find the maximum number of cross-over points using 5 then six lines and then find a constant pattern.





Correct tables

So now that I have found the maximum number of cross-over points I will now draw up a table, and then from this draw a graph to help me find a formula.

The table shows the new set of results.

No of lines	No of cross-over points
1	0
2	1
3	3
4	6
5	10
6	15

The table shows the constant pattern of the numbers also with the differences.

No of lines	No of cross-over points
1	0
	} 1
2	1
	} 2
3	3
	} 3
4	6
	} 4
5	10
	} 5
6	15

As you can see in the table above there is a constant pattern between each result.

Cross-over graph

This graph shows the results of the diagrams which I drew to show the maximum number of cross-over points when using a certain amount of lines.

Cross-over formula

No of lines	No of cross-over points
	1st difference
1	0
	} 1
2	1
	} 2
3	3
	} 3
4	6
	} 4
5	10
	} 5
6	15

If the second difference is any number above zero you need to half it to get the first part of your equation. You then need to look at your graph to see what the curve looks like. If the graph has a smooth curve like the one previously then the next part of the equation will be n^2 so then the equation will look like $\frac{1}{2}n^2$, but then because my second difference is 1 the equation will look like $0.5n^2$ because half of one is 0.5.

So now for the final part of the equation you need to incorporate your number of lines (n) into the first formula.

$n = 1$ so $n^2 = 1^2 = 1$

You need to then half it so you then have 0.5 .

So you then have the answer of 0.5. The number of cross-overs for one line is 0. Therefore to get from 0.5 to 0 you need to – 0.5 from the number of lines. This is the second part of your formula.

You would put an 'n' on to the end of this to symbolise it algebraically. Thus your final formula is

$$0.5n^2 - 0.5n$$

To test out this formula I decided to use n = 4 which I already have the answer to.

So, $0.5n^2 - 0.5n$: $4^2 = 16$, $0.5 \times 16 = 8$, $0.5 \times 2 = 2$

So 8 – 2 = 6 cross-overs. From looking at my table of results, this is correct.

So the formula to calculate the number of cross-over points is $0.5n^2 - 0.5n$.

Regions

At this point I am going to try and find the maximum number of regions using a certain number of lines. Hopefully while doing this I will discover a pattern. I will then look at the pattern and ultimately find a formula which corresponds to the number pattern, between the number of lines used and the number of regions.

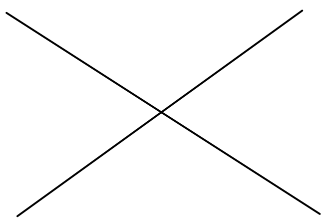
I looked at my diagrams which showed the maximum numbers of cross-overs. I then realised that because there is a maximum number of cross-over points there will be a maximum number of regions.

Aim: to find a relationship between the results from diagrams with maximum regions, using a certain number of lines. Ultimately, I will find a formula so that if I have a certain number of lines I can calculate how many regions there are.

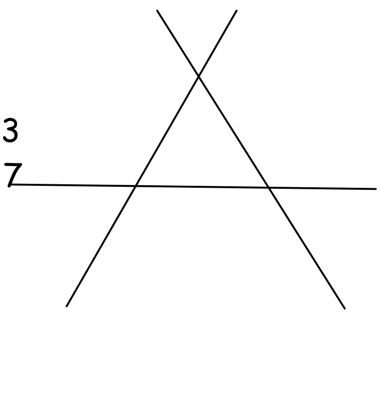
Regions diagrams

L = 1
R = 2

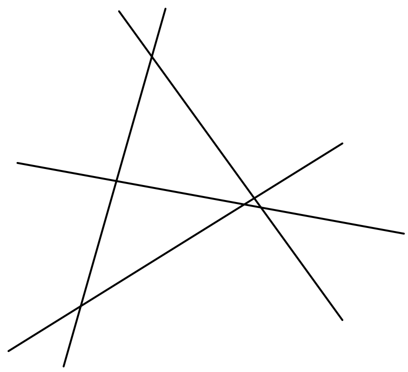
L = 2
R = 4



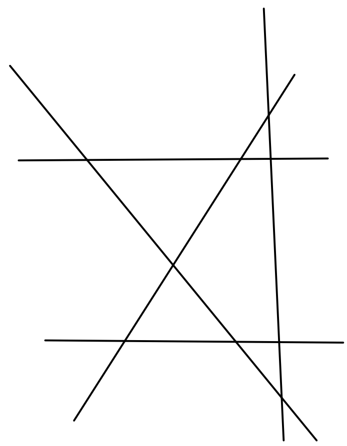
L = 3
R = 7



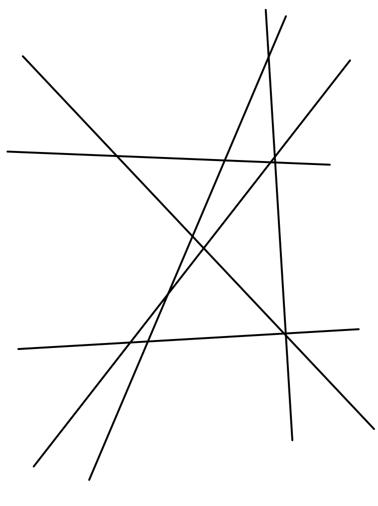
L = 4
R = 11



L = 5
R = 16



L = 6
R = 22



Those diagrams show the number of regions for a certain number of lines.

Tables

This table shows the results from the diagrams on the previous page of the regions for a certain number of lines.

No of lines	No of regions
1	2
2	4
3	7
4	11
5	16
6	22

Results table and finding the formula for the number of regions

No of lines	No of regions	1st difference
1	2	
		} 2
2	4	
		} 3
3	7	
		} 4
4	11	
		} 5
5	16	
		} 6
6	22	

If the second difference is any number above zero you need to half it to get the first part of your equation. You then need to look at the graph on the previous page to see what the curve looks like. If the graph has a smooth curve like the one previously then the next part of the equation will be n^2 so then the equation will look like $\frac{1}{2}n^2$, but then because my second difference is one in this case the equation will look like $0.5n^2$ because half of one is 0.5.

So now for the next of the equation you need to incorporate your number of lines (n) into the first part of your formula.

$n = 1$ so $n^2 = 1^2 = 1$

You need to then half it so you then have 0.5 .

So you then have the answer of 0.5. The number of regions for one line is two. Therefore to get from 0.5 to 2 you need to add half the number of lines ($\frac{1}{2}$) which is 0.5. So you should then have 1 because 0.5 + 0.5 = 1.

Obviously you need to get the answer of 2, because it shows in the table and also in the diagram that one line produces two regions.

So the formula should now look like this:

$$0.5n^2 - 0.5n + 1$$

To test out this formula I decided to use n = 4 which I already have the answer to.

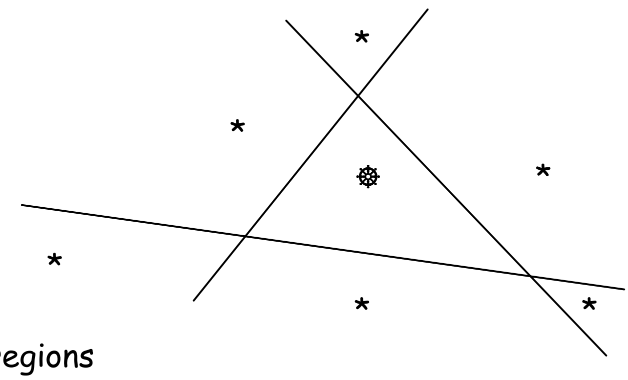
So, $0.5n^2 - 0.5n + 1$: $4^2 = 16$, $0.5 \times 16 = 8$, $0.5 \times 2 = 2$

So 8 + 2 = 10, 10 + 1 = 11 regions. From looking at my table of results, this is correct.

So the formula to calculate the number of regions points is
$0.5n^2 - 0.5n + 1$.

Open regions and closed

To look into this investigation in more depth I am not only going to look at regions I am going to look at open and closed regions too. I am going to look at my diagrams which I used to help me find a formula for the total number of regions. After looking at the diagrams I will draw up a table containing the number of open regions for a certain number of lines. Then a table for closed regions, then ultimately find formulas for each.



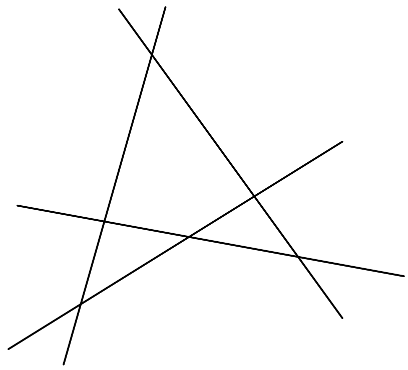
* = open regions
☸ = closed regions

Open and closed diagrams

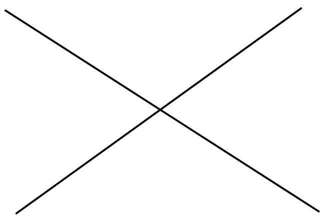
OR = open regions CR = closed regions

L = 1
OR = 2
CR = 0

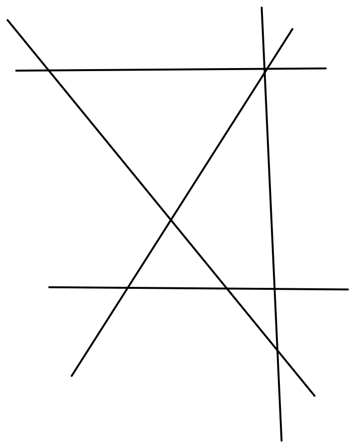
L = 4
OR = 8
CR = 3



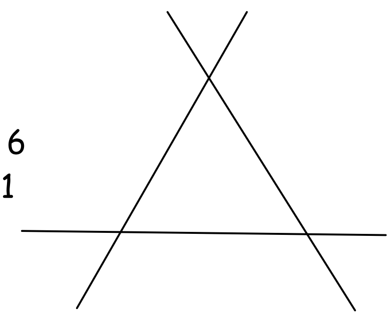
L = 2
OR = 4
CR = O



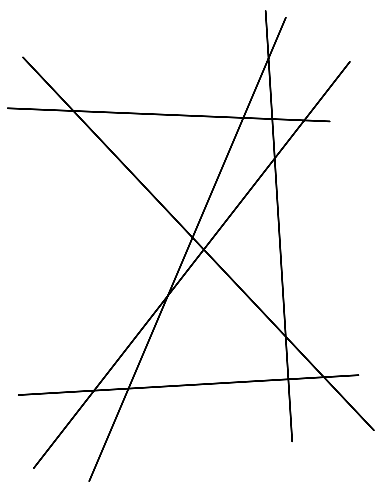
L = 5
OR = 10
CR = 6



L = 3
OR = 6
CR = 1



L = 6
OR = 12
CR = 10



Tables for open and closed regions

The table below show the results from the diagrams on the previous page.

No of lines	No of open regions	No of closed regions
1	2	0
2	4	0
3	6	1
4	8	3
5	10	6
6	12	10

Now that I have the results from my diagrams I will put them into the form of a graph.

Finding the formula

No of lines	No of open regions	1st difference
1	2	
		} 2
2	4	
		} 2
3	6	
		} 2
4	8	
		} 2
5	10	
		} 2
6	12	

Because the first difference is the same, the task of finding a formula will be much easier. It is very simple really even by just looking at the table. This type of number pattern is called "common difference". You just simply multiply the number of lines (n) by 2 to get the number of open regions.

Therefore the formula for open regions is $2n$ = open regions.

Finding the formula for closed regions

No of lines	No of regions	1st difference
1	0	
		} 0
2	0	
		} 1
3	1	
		} 2
4	3	
		} 3
5	6	
		} 4
6	10	

If the second difference is any number above zero you need to half it to get the first part of your equation. You then need to look at your graph to see what the curve looks like. If the graph has a smooth curve like the one previously, then the next part of your equation will be n^2. So then the equation will look like $\frac{1}{2}n^2$, but then because my second difference is 1 the equation will look like $0.5n^2$ because half of one is 0.5.

At this point this is where I have begun to have problems because both diagrams with 1 and two lines have no closed regions.

I have tried many times to find the next part of the equation but I don't seem to be able to find it.

Conclusion and relationship

From looking at my tables of results for cross-overs, regions, open regions and closed regions, there is a clear relationship between closed regions, regions and cross-overs because of their first and second differences. But open regions has a totally different pattern as you can see in the table. I have tried to find a formula for closed regions but this was very difficult to do. I found a formula for all of the other tasks but not for this one. It was very easy to find a formula for open regions because it was such a simple number pattern. It was slightly harder to find the formulae for the other tasks because their number patterns were more difficult. Looking at the first and second differences the numbers in the first differences contained the same number pattern +1, +2, +3, +4, +5; +2, +3, +4, +5, +6, +0, +1, +2, +3, +4, etc. You can see evidence of this in the tables. But the odd task out is open regions which has a different number pattern. If I were to do this task again I would as well as finding the formula and looking at the number patterns I would look more at the diagrams themselves, I would look more at the formula too.

Strand 1 Mark 5

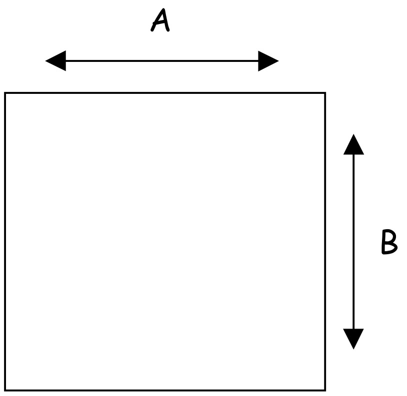
The candidate has drawn many diagrams as part of the work and is looking for the sequence, which indicates the maximum number of crossovers has been found. The candidate shows some insight into the problem as to how the maximum number of crossovers is generated. Later, the candidate extends the task to look at the relationship between the number of lines and open regions and also the number of lines and closed regions, and has introduced questions of their own.

Strand 2 Mark 5

The candidate states in the work the reason for the selection of tables and graphs is to try and find a formula connect the various variables in the task. Symbolic expressions are then presented to show some of these relationships.

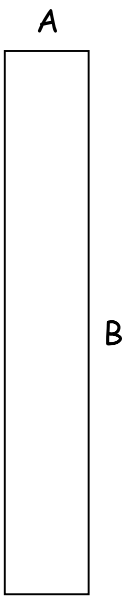
The work in the following section is taken from the 'Fencing Problem'.

I decided to start off my investigation by using different sized rectangles, making sure that the sides added up to 1000.



2A + 2B = 1000 metres
Area = A $\times$ B

Rectangle A:

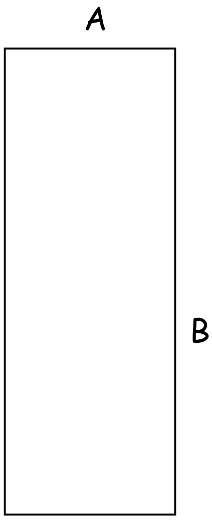


A = 50 m
B = 450 m

Area = A $\times$ B
Area = 50 $\times$ 450 = 22500

So this rectangle has an area of 22500 metres.

Rectangle B:

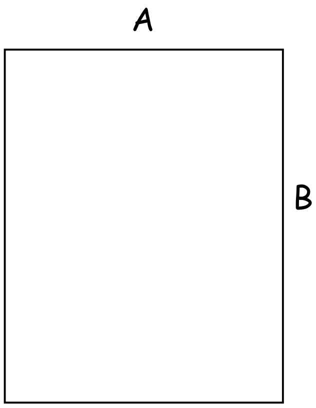


A = 100 m
B = 400 m

Area = A × B
Area = 100 m × 400 m = 40000 m

So this rectangle has an area of 40000 metres.

Rectangle C:

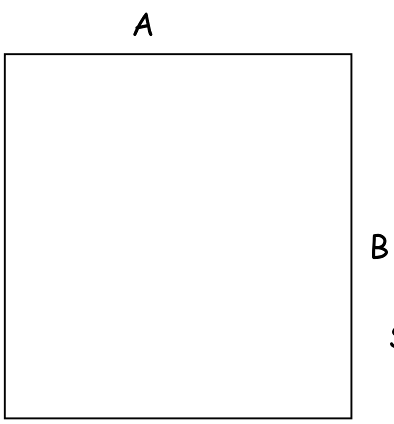


A = 150 m
B = 350 m

Area = A × B
Area = 150 m × 350 m = 52500 m

So this rectangle has an area of 52500 metres.

Rectangle D:

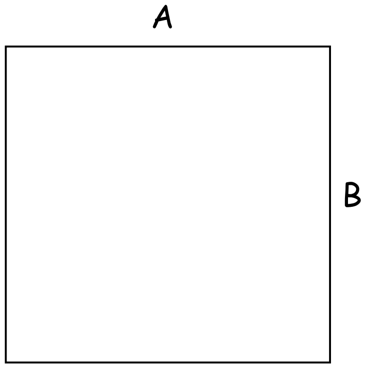


A = 200 m
B = 300 m

Area = A × B
Area = 50 m × 450 m = 60000 m

So this rectangle has an area of 60000 metres.

Rectangle E:



A = 250 m
B = 250 m

Area = A × B
Area = 250 m × 250 m = 62500 m

So this rectangle has an area of 22500 metres.

Having done these rectangles I decided to put my results into a table.

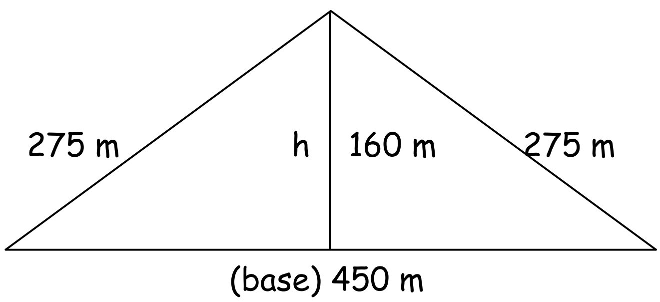
Shape	Side/m A	Side/m B	Area/m^2
A	50	450	22500
B	100	400	40000
C	150	350	52500
D	200	300	60000
E	250	250	62500

From my results I noticed that the biggest area possible is a square with sides of 250 each, making an area of 62500 $metres^2$.

Next I decided to find out the areas for different sized triangles. I started with picking the different sizes. I picked several different combinations, making sure again that they added up to 1000.

Triangle A:

e.g. this triangle has a base size of 450 metres and sides which are 275 metres each



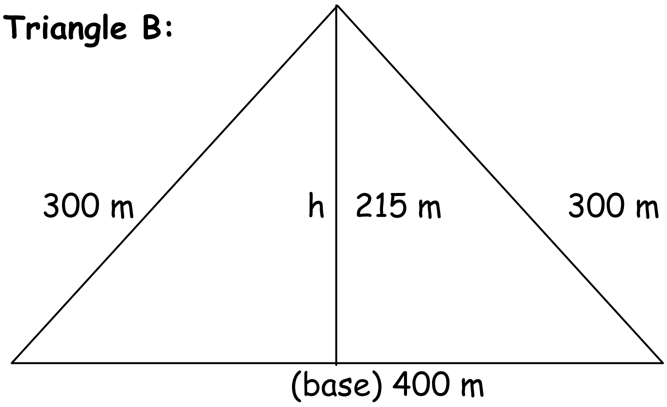
To work out the area of this triangle I measured the height (h) which was 160 metres. Then using the following formula:

$$\text{Area} = \frac{\text{Base x height}}{2}$$

Area = 450 m x 160 m = 72000,   72000 ÷ 2 = 36000

so the area of that triangle was 36000 metres. I picked different sizes and tried them.

Triangle B:

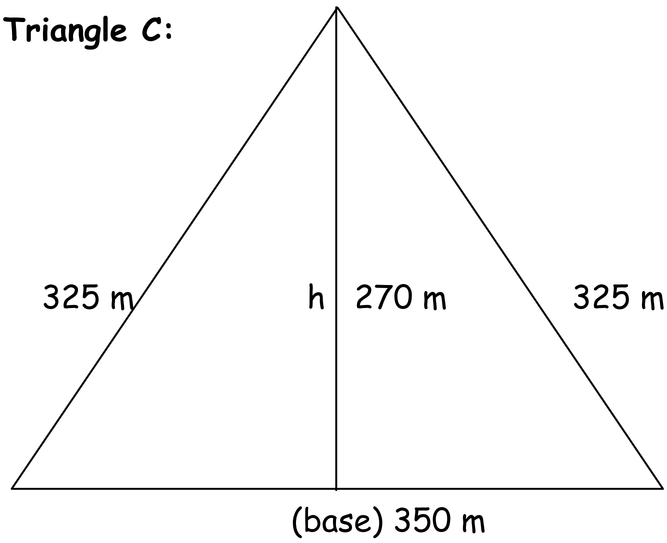


Area = $\frac{\text{Base x height}}{2}$

Area = 400 m x 215 m = 86000,   86000 ÷ 2 = 43000

The area of that triangle is = 43000 metres.

Triangle C:

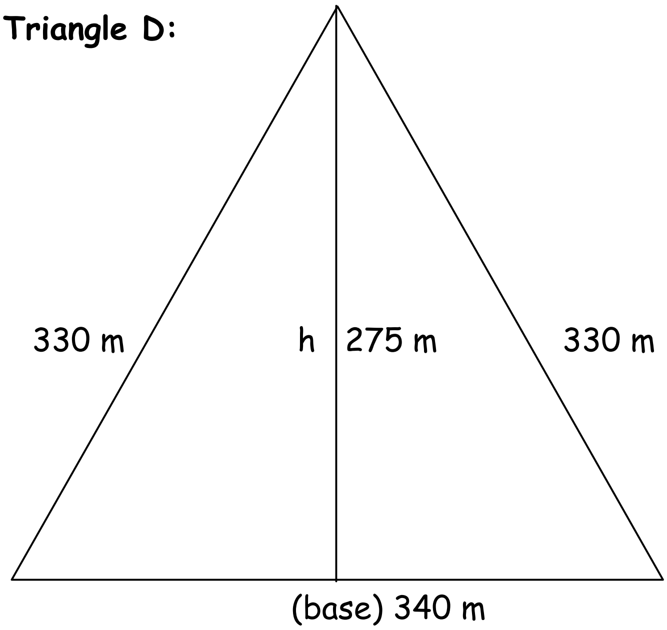


Area = $\frac{\text{Base x height}}{2}$

Area = 350 m x 270 m = 94500,   94500 ÷ 2 = 47250

The area of that triangle is = 47250 metres.

Triangle D:

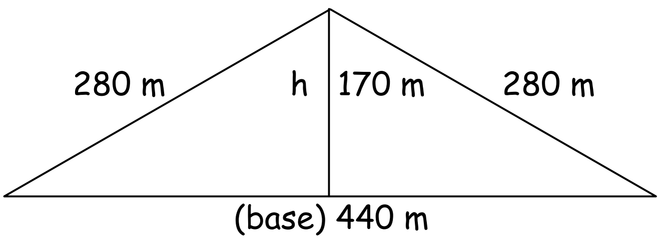


Area = $\frac{\text{Base x height}}{2}$

Area = 340 m x 275 m = 93500,   93500 ÷ 2 = 46750

The area of this triangle is = 46750 metres.

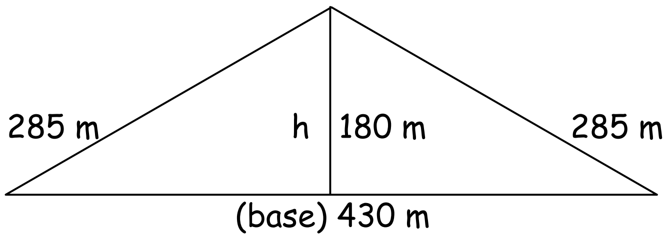
Triangle E:



Area = $\frac{\text{Base x height}}{2}$

This triangle has an area of: 37400 metres.

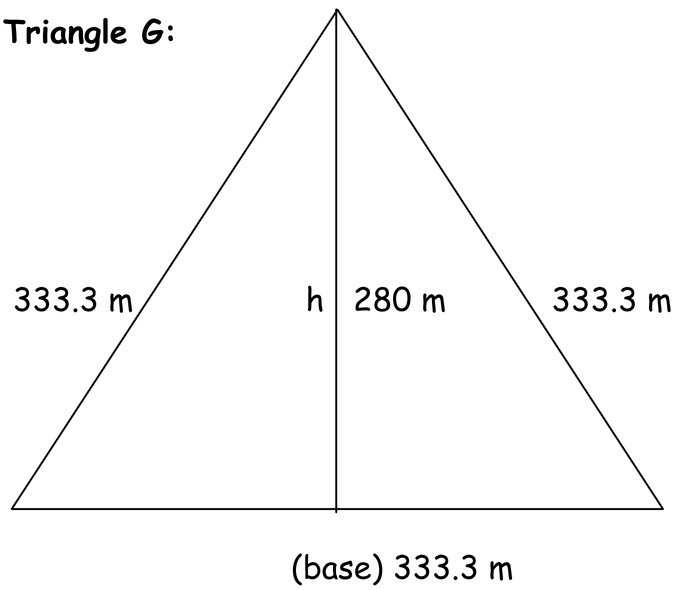
Triangle F:



$$Area = \frac{Base \times height}{2}$$

This triangle has an area of: 38700 metres.

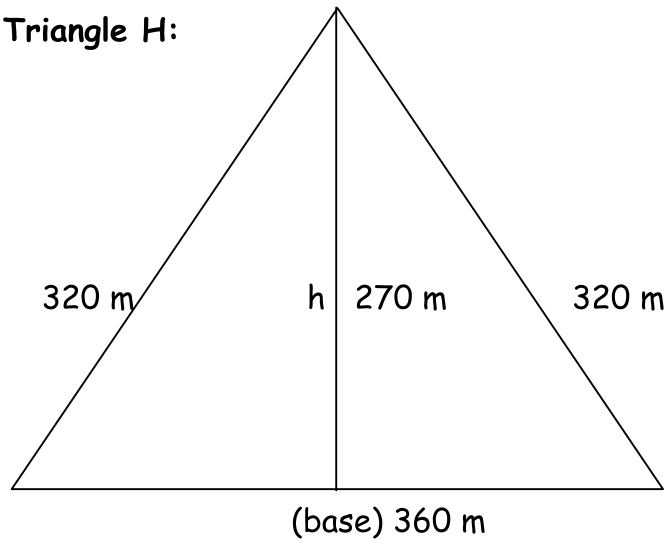
Triangle G:



$$Area = \frac{Base \times height}{2}$$

The area of this triangle is = 46662 metres.

Triangle H:



The area of this triangle is = 48600 metres.

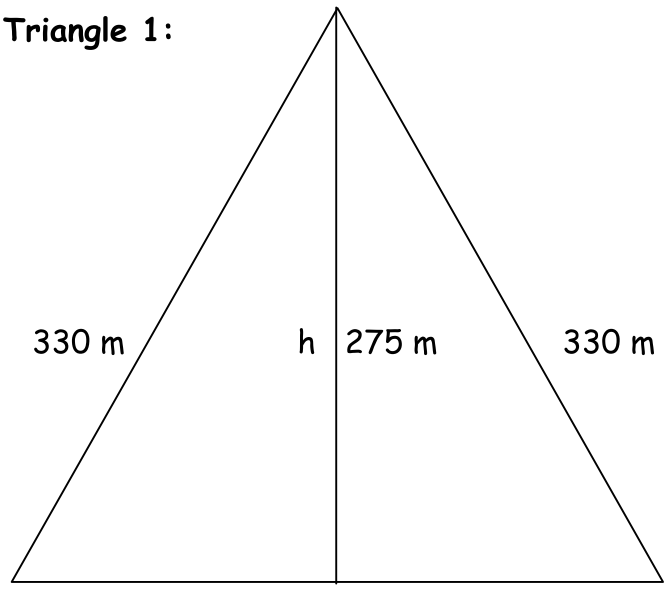
I decided to record my results from the triangles in a table.

Shape	Side A	Side B	Base	Area/m^2
A	275	275	450	36000
B	300	300	400	43000
C	325	325	350	47250
D	330	330	340	46750
E	280	280	440	37400
F	285	285	430	38700
G	333.3	333.3	333.3	46662
H	320	320	360	48600

all measurements in metres

From my table I realised that the triangles with the biggest areas would have bases of between 300 and 350. I thought that the triangle with the biggest area would be the one with 3 sides of 333.3 metres, however my table doesn't prove this due to an error either in the calculations or an error in the scale drawings. I decided to prove what I had predicted by drawing a few triangles and using Pythagoras rule to calculate the areas in the 300-350 range mathematically.

Triangle 1:



$(Hyp)^2 = A^2 + B^2$

Vertical height = $\sqrt{\text{side}\,A^2 - \left(\frac{\text{Base}}{2}\right)^2}$

$330^2 - 170^2$

108900 – 28900 = 80000

$\sqrt{80000}$

vertical height = 282.843

Triangle 2:

$(Hyp)^2 = A^2 + B^2$

350 base

325 side a

325 side B

vertical height = 325^2 – base ÷ 2 = 175^2

105625 – 30625 = 75000

$\sqrt{75000}$

vertical height = 273.861

Triangle 3:

$(Hyp)^2 = A^2 + B^2$

330 = base
335 = side A
335 = side B

vertical height = 335^2 – base ÷ 2^2 (165^2)
112225 – 27225 = 85000
$\sqrt{85000}$
v.h. = 291.548

Triangle 4:

$(Hyp)^2 = A^2 + B^2$

333.3 = base
333.3 = side A
333.3 = side B

vertical height = 333.3^2 – base ÷ 2^2 (166.665^2)
111108.89 – 27777.222 = 83331.668
$\sqrt{8331.668}$
v.h. = 288.675

Triangle 5:

$(Hyp)^2 = A^2 + B^2$

320 = base
340 = side A
340 = side B

vertical height = 340^2 – base ÷ 2^2 (160^2)
115600 – 25600 = 90000
$\sqrt{90000}$
v.h. = 300

Triangle 6:

$(Hyp)^2 = A^2 + B^2$

310 = base
345 = side A
345 = side B

vertical height = 345^2 – base ÷ 2^2 (155^2)
119025 – 24025 = 95000
$\sqrt{95000}$
v.h. = 308.221

Triangle 7:

$(Hyp)^2 = A^2 + B^2$

300 = base
350 = side A
350 = side B

vertical height = 350^2 – base ÷ 2^2 (150^2)
122500 – 22500 = 100000
$\sqrt{100000}$
vertical height = 316.228

After this I found out the area of the triangles by multiplying the base by the vertical height and dividing by 2. I decided to put the results of this into a table.

Triangle	Base	Height	Area
7	300	316.228	94868.4
6	310	308.221	47774.225
5	320	300	48000
3	330	291.548	48105.42
4	333.33	288.675	48112.019
1	340	282.843	48083.31
2	350	273.861	47925.675

so from my table I noticed that the triangle with the biggest area is one with a base of 333.33 and sides of 333.33, making it an equilateral triangle.

To prove the biggest area was from a 333.33 sides triangle I put the bases from 300 – 380 onto a table:

Base	Sides	Height	Area
300.00	350.00	316.23	47,434.50
310.00	345.00	308.22	47,774.25
320.00	340.0	300.00	48,000.00
330.00	335.00	291.55	48,105.42
333.33	333.33	288.68	48,112.02
340.00	330.00	282.84	48,083.31
350.00	325.00	273.86	47,925.68
360.00	320.00	264.57	47,623.52
370.00	315.00	254.95	47,165.93
380.00	310.00	244.95	46,540.31

This shows that the area sixes "peak" at 333.33 (drop after it, rise before it). It also shows the biggest area is within a triangle with sides of 333.33.

Next I decided to find out the area for a circle. There is only one circle that can be done, and I started it by working out the radius.

To work out the radius I used: $A = \pi \times R \times R$

$C = 2\pi R$
$1000 = 3.142 \times 2R$
$1000 = 6.284 \times R$
$1000 \div 6.284 = R$

$R = 159.134$

Now that I have worked out the radius i used the formula $A = \pi R^2$ to work out the area.

$Area = 3.142 \times (159.134 \times 159.134) = 79566.8$

so the area from the circle was 79566.8 metres.

Strand 1 Mark 5

The candidate has considered the four-sided shape and has now moved on to consider triangles. Adopting this approach should enable the candidate to move towards a fuller solution to the task.

The work for the following section has been taken from 'T-Totals'.

The relationship between T-total and T-number

8 × 8 grid:

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

Let's say the size of the grid is "*m* ".

1. Let's say x = T-number

11	12	13		x –17	x -16	x -15
	20		=		x - 8	
	28				x	

The T-total is: x + x – 8 + x – 15 + x – 16 + x – 17.
If we put those numbers together this is what we would get → 5x – 56

If we translate the T-shape into different position:

45	46	47		x –17	x -16	x -15
	54		=		x - 8	
	62				x	

The T-total is: x – 17 + x – 16 + x – 15 + x – 8 + x. This is the same one as we have got above. If we add them together again, we will get the same answer again: 5x – 56.

The solutions above show that it doesn't matter how we translate the T-shape in the 8 × 8 grid into different positions. We would still get the same answer:
5x – 56.

9 × 9 grid

1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81

11	12	13		x −19	x −18	x −17
	21		=		x − 9	
	30				x	

= x + x − 9 + x − 17 + x − 18 + x − 19
= 5x − 63

50	51	52		x −19	x −18	x −17
	60		=		x − 9	
	69				x	

= x + x − 9 + x − 17 + x − 18 + x − 19
= 5x − 63

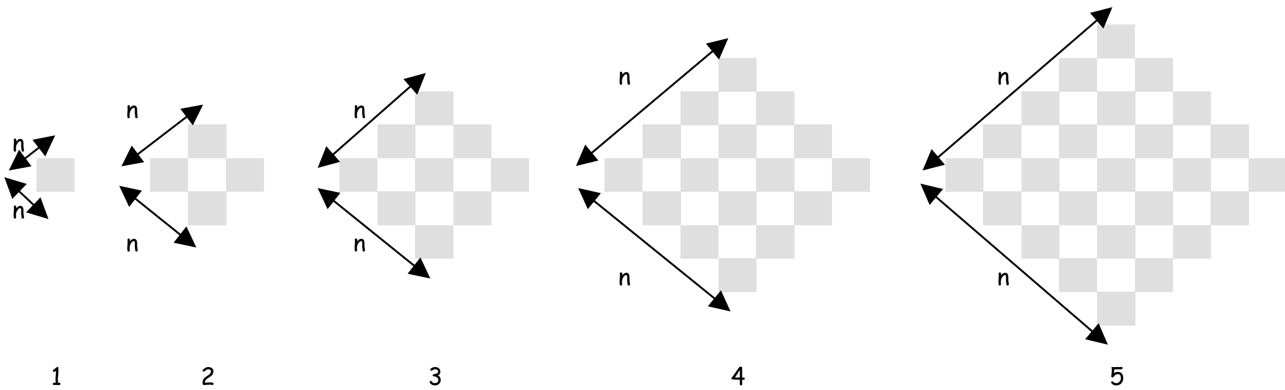
It doesn't matter how we translate the T-shape into different positions, we would still get 5x − 63.

Strand 3 Mark 5

The candidate shows some understanding of the structure of the T-shapes within different size grids and thus moves from experimental to theoretical explanation. The adding together of the algebraic terms clearly justifies where the $5n$ – 56 etc came from.

Another example is taken from the task 'Borders'

Borders

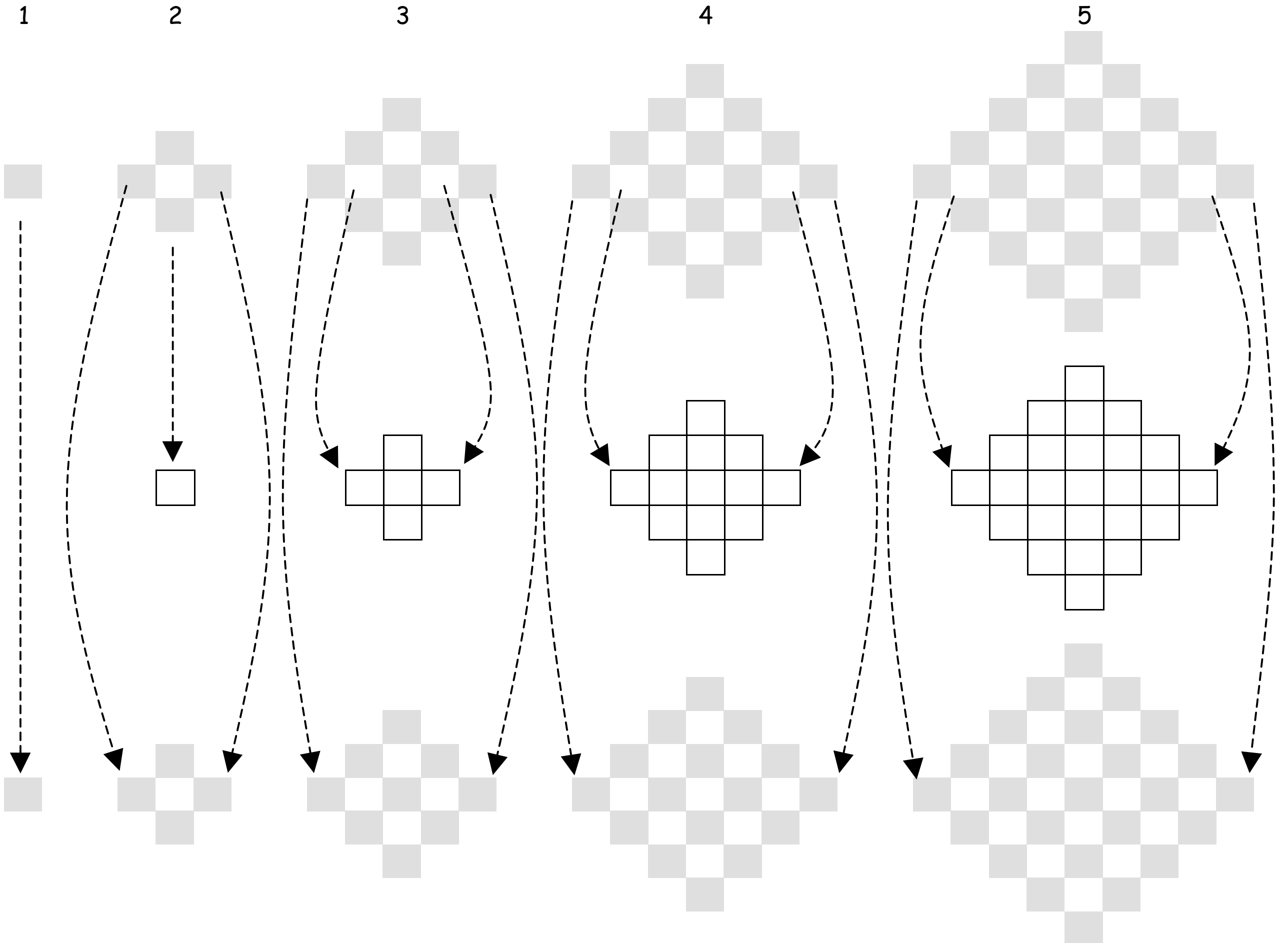


represents those belonging to n^2

represents those belonging to $(n - 1)^2$

If each of the above diagrams were rotated 45% it would be clear that each shape is symbolical to a square. Make it that n is the number of squares on any one edge of any cross shape. You can see that the number is equal to the term number (n). When n is squared (as in the above diagrams), you can see that there are still squares remaining. If you analyse each cross shape individually, you can see that the remaining white squares are exactly the same as the crossed squares are exactly the same as the crossed squares in the previous cross shape.

i.e.



Now that it is clearer to see the obvious connection, the justification will be easier to prove.

Look at the fifth cross shape (overleaf). The squares that are shaded represent n^2. The remaining squares are equal to those that are criss-crossed in the previous cross shape (which in the fourth cross shape represent n^2). Because these make up the fifth cross shape, it is true to say that

$n^2 + (n - 1)^2 = S$ (number of cubes)

-1 because it is the previous term

When we expand this equation…

$n^2 + (n - 1)^2 = n^2 + (n - 1)(n - 1) = n^2 + n^2 - n - n + 1 = 2n^2 - 2n + 1$

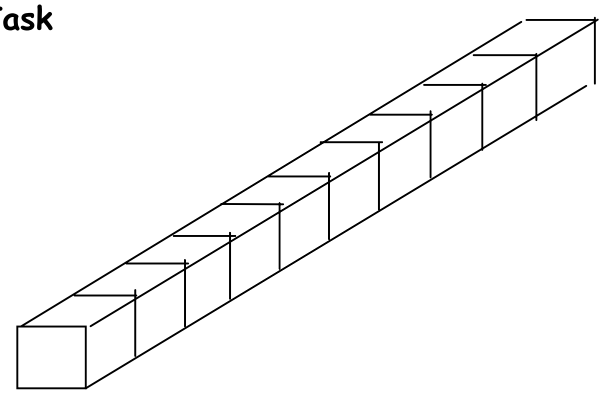
We are left with the formula that we obtained earlier.

Strand 3 Mark 5

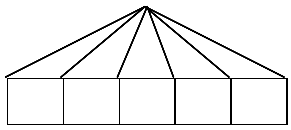
The candidate has clearly referred to the structure of the task to explain the expression obtained. This approach looks initially at the geometry of the shape and then algebraically.

Another example is taken from the task ‘Hidden Faces’

Hidden Faces Task



The formula that I have found is 3N – 2 for the hidden faces (N = number of cubes).



no of faces

There are n faces hidden on the table. There are 2 faces in the gaps and there are (n – 1) gaps.

$\therefore$ Hidden faces $= 2(n - 1) + n$

$= 2n - 2 + n$

$= 3n - 2$

Strand 3 Mark 5

The candidate has obtained the general formula for the number of hidden faces. She has then looked at the structure of the situation to justify where the coefficient of ‘*n*’ and the ‘–2’ originates.

This gains the award of Mark 5 in Strand 3.

MARK 6

The work in the section below is taken from the task 'The Fencing Problem.'

So far I've discovered that the less sides a shape has, the smaller its area will be. I predict that the circle will have the biggest area, as it is made up of an infinite number of sides. I'm now going to work out the area of a circle and see if my prediction is right.

Circles

I know the circumference of the circle should be 1000 m. To find the diameter of the circle, and the area, I'll have to turn around the equation $C = \pi d$ which you use to find the circumference of a circle.

The equation is $d = \frac{C}{\pi}$.

I know what the circumference and π is so I can find out what the diameter is.

$d = \frac{C}{\pi} = \frac{1000}{3.14159} = 318.309862\ldots = 318.3$ m (1 d.p.)

I can now find the area of the circle with the equation $A = \pi r^2$. The radius will be half the diameter = 159.15 m.

$$\begin{aligned} \text{Area } &= \pi r^2 \\ &= \pi \times 159.15 \times 159.\ 15 \\ &= 79572.52853 \\ &= 79572.5 \text{ m}^2 \end{aligned}$$

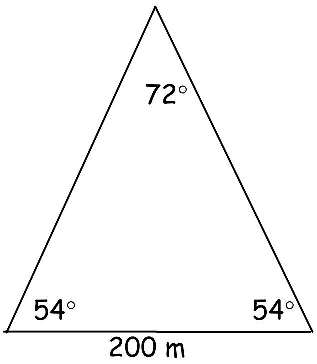
My prediction was right. The circle has the biggest area, as it is made up of an infinite number of sides. I'm going to carry on testing it up to a 20-sided shape. I'll then put my results in a table and see if there is a pattern. To find the height of the shape I'm going to use trigonometry.

5 sided polygon

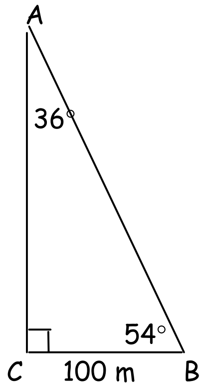
The edges of the shape are equal. So 1000 m ÷ 5 = 200 m.

A five-sided polygon is made up of 5 triangles. To find the area of the shape, I'm going to find the height of one triangle, then the area of it, and then times it by five.

One of five triangles:



Half of triangle:



$\tan 54 = \frac{AC}{100}$ $\quad AC = \tan 54 \times 100$

$= 137.638192$

$= 137.6$ m

$\therefore$ Area of triangle $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 200 \times 137.6$

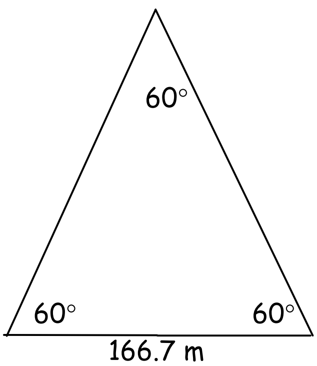
$= 13760 \text{ m}^2$

There a 5 triangles in shape = $5 \times 13760 = 68{,}800 \text{ m}^2$

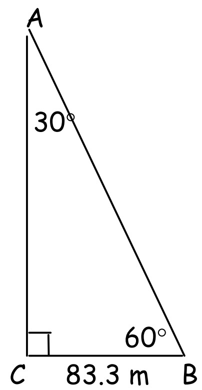
6 sided polygon

Edges of shape = $1000 \div 6 = 166.7$ m

One of six triangles:



Half of triangle:



$\tan 60 = \frac{AC}{83.35}$ $\quad AC = \tan 60 \times 83.35$

$= 144.3664348\ldots$

$= 144.4$ m

∴ Area of triangle $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 166.7 \times 144.4$

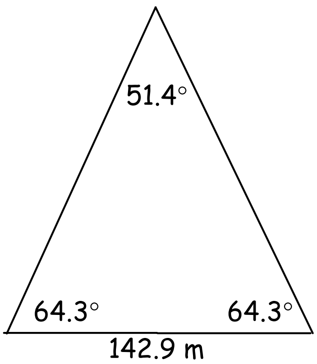
$= 12035.74\ m^2$

There are 6 triangles in shape = $6 \times 12035.74 = 72,214.44\ m^2$

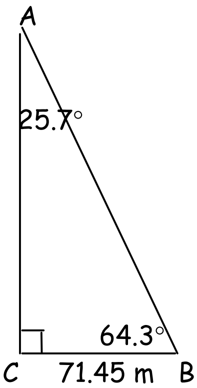
7 sided polygon

Edges of shape = $1000 \div 7 = 142.9$ m

One of seven triangles:



Half of triangle:



$\tan 64.3 = \frac{AC}{71.45}$ $\quad AC = \tan 64.3 \times 71.45$

$= 148.54621341...$

$= 148.5$ m

∴ Area of triangle $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 142.9 \times 148.5$

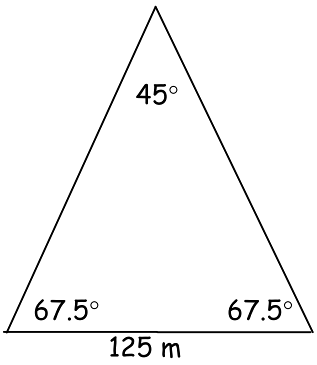
$= 10610.325\ m^2$

There are 7 triangles in shape = $7 \times 10610.325 = 74274.275\ m^2$

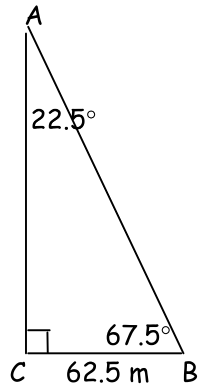
8 sided polygon

Edges of shape = 1000 ÷ 8 = 125 m

One of eight triangles:



Half of triangle:



$\tan 67.5 = \dfrac{AC}{62.5}$ $\qquad$ $AC = \tan 67.5 \times 62.5$

$= 150.88.3$

$= 150.9$ m

$\therefore$ Area of triangle $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 125 \times 150.9$

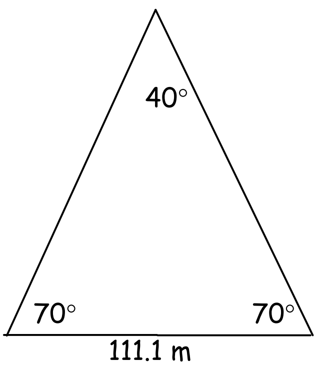
$= 9431.25\ m^2$

There are 8 triangles in shape = $8 \times 9431.25 = 75450\ m^2$

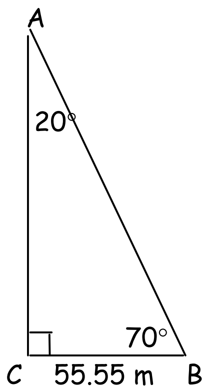
9 sided polygon

Edges of shape = 1000 ÷ 9 = 111.1 m

One of nine triangles:



Half of triangle:



$\tan 70 = \dfrac{AC}{55.55}$ $AC = \tan 70 \times 55.55$

$= 152.62237$

$= 152.6$ m

$\therefore$ Area of triangle $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 152.6 \times 144.4$

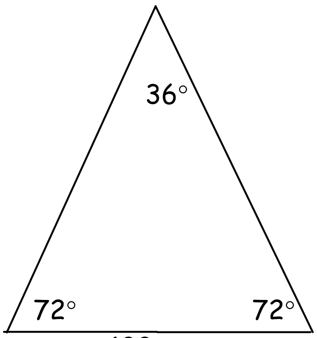
$= 8476.93 \text{ m}^2$

There are 9 triangles in shape = $9 \times 8476.93 = 76292.37 \text{ m}^2$

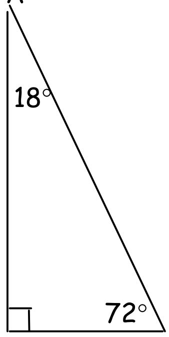
10 sided polygon

Edges of shape = $1000 \div 10 = 100$ m

One of ten triangles:



Half of triangle:



$\tan 72 = \dfrac{AC}{50}$ $AC = \tan 72 \times 50$

$= 153.884\ldots$

$= 153.9$ m

$\therefore$ Area of triangle $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 100 \times 153.9$

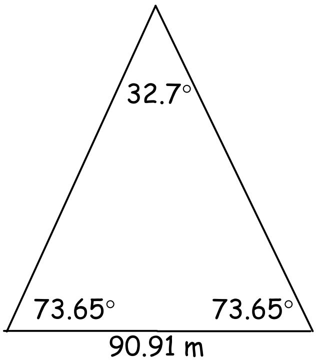
$= 7695 \text{ m}^2$

There are 10 triangles in shape = $10 \times 7695 = 76950 \text{ m}^2$

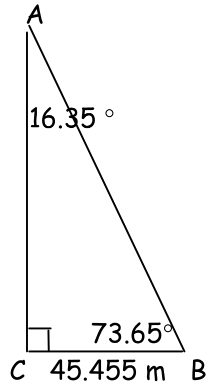
11 sided polygon

Edges of shape = 1000 ÷ 11 = 90.91 m

One of eleven triangles:



Half of triangle:



$$\tan 73.65 = \frac{AC}{45.455} \qquad AC = \tan 73.65 \times 45.455$$

$$= 154.9419\ldots$$

$$= 154.9 \text{ m}$$

$\therefore$ Area of triangle $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 90.91 \times 154.9$

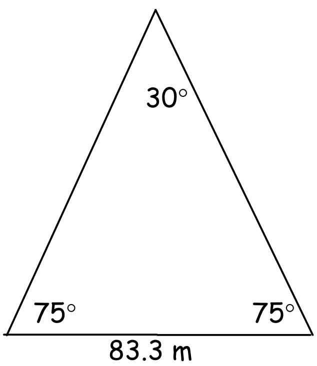
$= 7040.9795 \text{ m}^2$

There are 11 triangles in shape = 11 x 7041 = 77451 m^2

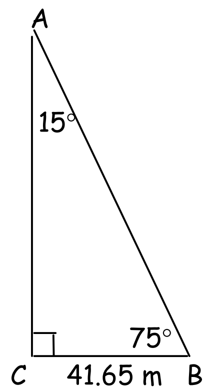
12 sided polygon

Edges of shape = 1000 ÷ 12 = 83.3 m

One of six triangles:



Half of triangle:



$$\tan 75 = \frac{AC}{41.65} \qquad AC = \tan 75 \times 41.65$$

$$= 155.439\ldots$$

$$= 155.4 \text{ m}$$

$\therefore$ Area of triangle $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 83.3 \times 155.4$

$= 6472.41 \text{ m}^2$

There are 12 triangles in shape = $12 \times 6472.41 = 77668.92 \text{ m}^2$

Strand 1 Mark 6

The candidate has previously covered three and four sided shapes and has found, with justification, that the maximum area obtained is the regular case. The candidate now reflects upon this and moves on to consider regular polygons based upon the evidence already produced. Other techniques are now used to find the area of several polygons. This is worthy of mark 6 in the first strand.

Strand 2 Mark 6

The candidate has consistently used trigonometry in the work. This is therefore awarded a mark of 6 in strand 2 as 'consistent use of symbolism.' In this case, the symbolism is trigonometry (refer to elaboration document).

Another example of Strand 2 Mark 6 can be seen in the extract below from 'Beyond Pythagoras':

Introduction

I was set a problem. It was to investigate Pythagorean Triples. The task was to find and prove formulas for finding these Pythagorean triples. I hope to find these formulas which will give me the nth Pythagorean triple and the Area and Perimeter of it.

I am starting by proving the simple triples that I already know, such as 5, 12, 13, as these numbers satisfy the condition of the Pythagoras Theorem i.e. $(\text{smallest number})^2 + (\text{middle number})^2 = (\text{largest number})^2$ because:
$5^2 = 5 \times 5 = 25$
and
$12^2 = 12 \times 12 = 144$.

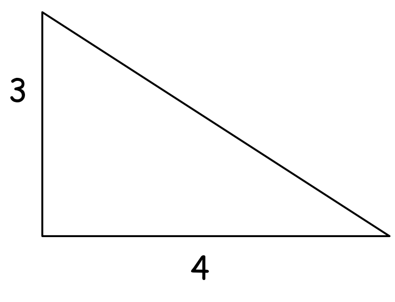
So therefore when I add the numbers together, 25 + 144 = 169.

This sum gives me the answer which is the largest side squared. Therefore, if calculate the square root of 169, I should get the largest side which I already know is 13; $\sqrt{169} = 13$.

So, now I know that this is a triple, because the largest side is an integer.

Pythagorean Theorem:
$5^2 + 12^2 = 13$

The numbers 3, 4, 5 also fit into the equation and would be drawn as a right angled triangle like this:



The numbers work because:
$3^2 = 3 \times 3 = 9$
and
$4^2 = 4 \times 4 = 16$

So when I add them together I get; 16 + 9 = 25 which is the largest side squared. Therefore when I calculate the square root of 25 I should get 5; $\sqrt{25} = 5$.

This is therefore proof that the numbers 3, 4, 5 fit into the equation $3^2 + 4^2 = 5^2$.
Now I am going to attempt to find a formula for the shortest side when it is an odd number using n for the diagram number.

n	shortest side	difference 1
1	3	2
2	5	2
3	7	

This difference column tells me I have to add on 2 every time. So I am going to try timesing n by 2.

n	shortest side
1 x 2	3
2 x 2	5
3 x 2	7

So I have found that if I times n by 2 ,I will still have to add I to get the shortest side.
The formula is therefore: $\underline{2n + 1}$
n

n	shortest side
1 x 2 + 1	3
2 x 2 + 1	5
3 x 2 + 1	7

I am now going to attempt to find the formula for n and the middle side, when the shortest side is an odd number.

n	middle side	difference 1	difference 2
1	4	8	
2	12	12	4
3	24		

The above helps me because when I add the second difference column to the first it gives me the number which I have to add on to the last number in the sequence of the middle side. For example;

n	middle side	difference 1	difference 2
1	4	8	4
2	12	12	
3	24	24	

So 4 + 12 + 24 = 40

This means that when n is 4 the middle side is 40.

So first I am going to times n by 4

n	middle side
1 x 4 = 4	4
2 x 4 = 12	12
3 x 4 = 24	24
4 x 4 = 16	40

This is obviously incorrect.

I will now square n first and then add 2 times n

n	middle side
1 x 1 + 2 x 1 = 3	4
2 x 2 + 2 x 2 = 8	12
3 x 3 + 2 x 3 = 15	24
4 x 4 + 2 x 4 = 24	40

I am not quite there.

This time I will times n by 2 then add 2 times n.

n	middle side
1 x 1 x 2 + 2 x 14	4
2 x 2 x 2 + 2 x 2 = 12	12
3 x 3 x 2 + 2 x 3 = 24	24
4 x 4 x 2 + 2 x 4 = 40	40

So, 1 have found the formula for n and the middle side. for example;
When n is 100, the middle side is 20,200 because

$2 \times 100^2 + 2 \times 100 = 20{,}200$

So the formula is

$2n^2 + 2n$

I have noticed a pattern in the short table which I was given to start with, between the middle side and the longest side. This enables me to work out the formula for the longest side, when the shortest side is an odd number.

n	middle side	longest side
1	4	5
2	12	13
3	24	25

As you can see from the table above the longest side is always I longer than the middle side, so the formula must be the same as for the middle side but with +1 added at the end.

$2n^2 + 2n + 1$

To find the formula for the perimeter when the shortest side is an odd number I have to add the formula for finding the shortest side, the formula for finding the middle side and the formula for finding the longest side together.

So,

$2n + 1 + 2n^2 + 2n + 2n^2 + 2n + 1$

I then add these together which gives me

$4n^2 + 6n + 1 + 1$

and finally I add the two l's together which gives me the formula for the perimeter:

$4n^2 + 6n + 2$

In order for me to find the formula for the area, l have to times the formula for the shortest side by the formula for the middle side and divide them by two. So,

$(2n + 1)\ (2n^2 + 2n)$

$4n^3 + 4n^2 + 2n^2 + 2n$

If I then bring this all together it gives me a result of

$$\frac{4n^3 + 6n^2 + 2n}{2}$$

$= 2n^3 + 3n^2 + n$

Strand 2 Mark 6

The candidate has used correct symbolism to establish the algebraic expressions for the sides of right-angled triangles. These are then used to find the expressions for the area/perimeter of the triangles. The variables are defined and the work is correct. Therefore a mark of 6 in strand 2 is awarded.

A further example of **Strand 2 Mark 6** can be seen in the extract below from the task 'T-totals':

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

T-number	T-total
16	31
18	41
20	51
39	146
38	143
40	151

41 – 31 = 10

10 N = 10(16)
= 160

160 – 31

10N = 10(16)
= 160

Trying out the formula in Q2
5n – 63 = 5(16) – 63
= 17

17 is not the T-total. The formula for Q1 doesn't work for a different grid size.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

T-number	T-total
12	25
19	60
23	75

25 – 12 = 13
60 – 19 = 41
75 – 22 = 53

41 – 13 = 28
53 – 41 = 12

60 – 25 = 35
75 – 60 = 15

1	2	3
	7	
	12	

2	3	4
	8	
	13	

3	4	5
	9	
	14	

T-number	T-total
16	25
17	30
18	35

The difference between the T-totals whose T-number comes one after each other has a difference of 5.
Using the idea from the original formula (5n – 63) we put an "n" beside the 5.

$$12 + [12 - (12 - 1)] + [12 - (12 - 2)] + [12 - (12 - 3)] + [12 - (12 - 7)]$$

$$= 12 + (12 - 11) + (12 - 10) + (12 - 9) + (12 - 5)$$

$$= 12 + 1 + 2 + 3 + 7$$

$$= 25$$

25 is also the T-total.

Doing the same thing again to make sure it really works,

$$19 + [19 - (19 - 8)] + [19 - (19 - 9)] + [19 - (19 - 10)] + [19 - (19 - 14)]$$

$$= 19 + (19 - 11) + (19 - 10) + (19 - 9) + (19 - 5)$$

$$= 19 + 8 + 9 + 10 + 14$$

$$= 60$$

60 is also the T-total.

The answer from the T-number minimising all the other numbers from the T-shape. It is always (example) $19 + (19 - \underline{11}) + (19 - \underline{10}) + (19 - \underline{9}) + (19 - \underline{5})$ numbers in those places that are the same but only for a 5 by 5 grid.

Using n to replace the T-number,

$$n + [n - (19 - 8)] + [n - (19 - 9)] + [n - (19 - 10)] + [n - (19 - 14)]$$

$= \quad n + (n - 11) + (n - 10) + (n - 9) + (n - 5)$

$= \quad n + n - 11 + n - 10 + n - 9 + n - 5$

$= 5n - 35$

The formula for a 5 by 5 grid is $5n - 35$.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

T-number	T-total
14	28
17	43
33	123

Doing the formula like the 5 by 5 grid

$14 + [14 - (14 - 1)] + [14 - (14 - 2)] + [14 - (14 - 3)] + [14 - (14 - 8)]$

$= \quad 14 + (14 - 13) + (14 - 12) + (14 - 11) + (14 - 6)$

$= \quad 14 + 1 + 2 + 3 + 8$

$= \quad 28$

28 is also the T-total.

Using "n" instead of the T-number,

$n + [n - (14 - 1)] + [n - (14 - 2)] + [n - (14 - 3)] + [n - (14 - 8)]$

$= \quad n + n - 13 + n - 12 + n - 11 + n - 6$

$= \quad 5n - 40$

Checking my formula,

$5n - 42 \quad = 5(17) - 42$

$= 85 - 42$

$= 43$

The formula for a 6 by 6 square grid is $5n - 42$.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

T-number	T-total
16	31
33	116
44	171

Using the idea of how the formula was found in all the grids before:

$$16 + [16 - (16 - 1)] + [16 - (16 - 2)] + [16 - (16 - 3)] + [16 - (16 - 9)]$$

$$= 16 + (16 - 15) + (16 - 14) + (16 - 13) + (16 - 7)$$

$$= 16 + 1 + 2 + 3 + 9$$

$$= 31$$

31 is also the T-total.

Using "n" instead of the T-number,

$$n + [n - (16 - 1)] + [n - (16 - 2)] + [n - (16 - 3)] + [n - (16 - 9)]$$

$$= n + (n - 15) + (n - 14) + (n - 13) + (n - 7)$$

$$= n + n - 15 + n - 14 + n - 13 + n - 7$$

$$= 5n - 49$$

Checking the formula:

$$5n - 49 = 5(33) - 49$$

$$= 165 - 49$$

$$= 116$$

116 is the T-total.

The formula for a 7 by 7 square grid is $5n - 49$.

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

T-number	T-total
18	34
39	139
51	199
62	254

Using the idea of how the formula was found in all the grids before:

18 + [18 – (18 – 1)] + [18 – (18 – 2)] + [18 – (18 – 3)] + [18 – (18 – 10)]

= 18 + (18 – 17) + (18 – 16) + (18 – 15) + (18 – 8)

= 18 + 1 + 2 + 3 + 10

= 34

34 is the T-total.

Using "n" instead of the T-number,

n + [n – (18 – 1)] + [n – (18 – 2)] + [n – (18 – 3)] + [n – (18 – 10)]

= n + (n – 17) + (n – 16) + (n – 15) + (n – 8)

= n + n – 17 + n – 16 + n – 15 + n – 8

= 5n – 56

Checking the formula:

5n – 49 = 5(33) – 49

= 165 – 49

= 116

116 is the T-total.

The formula for a 8 by 8 grid is 5n – 56.

Grid sizes	Formulas
5 by 5	5n – 35
6 by 6	5n – 42
7 by 7	5n – 49
8 by 8	5n – 56

In all formulas, they all begin with 5n. The endings of the formulas are linked together by the number 7.

$5n \pm 7$: the sign could be "+" or "–". I will try "–" first because all of the formulas have "–" instead of "+".

6 by 6 grid

20	21	22
	27	
	33	

$$\begin{aligned} 5n - 7 &= 5(33) - 7 \\ &= 165 - 7 \\ &= 158 \end{aligned}$$

The formula doesn't work.

The number behind the n, e.g. 5n – **35**, give the same number as the grid size number if divided by 7.

$35 \div 7 = 5$	5 by 5
$42 \div 7 = 6$	6 by 6
$49 \div 7 = 7$	7 by 7
$56 \div 7 = 8$	8 by 8

so the formula should be: 5n – 7 x grid size.

To test my formula,

$$\begin{aligned} &5n - 7 \times \text{grid size} \\ &= 5(62) - (7 \times 8) \\ &= 310 - (7 \times 8) \\ &= 310 - 56 \\ &= 254 \end{aligned}$$

7 by 7 grid

45	46	47
	54	
	62	

T-total = 254

0°

n + (n – 1) + (n – 2) + (n – 3) + (n – 9)

= n + n – 1 + n – 2 + n – 3 + n – 9

= 5n – 15

Not right because this answer should be 5n – 49 like the answer for the 7 by 7 grid in question 2.

Trying another way, not changing the T-number to n:

0°

45	46	47
	54	
	62	

16 – 1	16 – 2	16 – 3
	16 – 9	
	16	

Using n instead of the T-number,

n + [n – (16 – 1)] + [n – (16 – 2)] + [n – (16 – 3)] + [n – (16 – 9)]

= n + (n – 15) + (n – 14) + (n – 13) + (n – 7)

= n + n – 15 + n – 14 + n – 13 + n – 7

= 5n – 49

This is the right answer.

Using this way for all other angles,

90°

		19
24	25	26
		33

180°

	23	
	30	
36	37	38

90°

		24 – 19
24	24 – 25	24 – 26
		24 – 33

180°

	23	
	23 – 30	
23 – 36	23 – 37	23 – 38

Using n instead of the T-number,

90°

$$n + [n - (24 - 25)] + [n - (24 - 26)] + [n - (24 - 33)] + [n - (24 - 19)]$$

$$= n + (n - -1) + (n - -2) + (n - -9) + (n - 5)$$

$$= n + n + 1 + n + 2 + n + 9 + n - 5$$

$$= 5n + 7$$

180°

$$n + [n - (24 - 36)] + [n - (23 - 37)] + [n - (23 - 38)] + [n - (23 - 16)]$$

$$= n + (n - -13) + (n - -14) + (n - -15) + (n - 7)$$

$$= n + n + 13 + n + 14 + n + 15 + n - 7$$

$$= 5n + 35$$

Grid size	formulas
	① ② ③ ④
5 by 5	n + (n – 11) + (n – 10) + (n – 9) + (n – 5)
6 by 6	n + (n – 13) + (n – 12) + (n – 11) + (n – 6)
7 by 7	n + (n – 15) + (n – 14) + (n – 13) + (n – 7)
8 by 8	n + (n – 17) + (n – 16) + (n – 15) + (n – 8)
9 by 9	n + (n – 19) + (n – 18) + (n – 17) + (n – 9)

There is a pattern: ② – ④ always equals the grid size.

All the ①s which (their grid sizes come one after the other) come after one another is made by the T-number (n) at the front and 11 from the last number, e.g. 13 and you always get 2.

Put the gird size (G) in the place of the number behind the T-number, e.g. n – G.

To get ② from ① you have to – 1.
To get ② from ③ you have to + 1.

So using n instead of T-number,

$$n + (n - G) + (n - 2G + 1) + (n - 2G) + (n - 2G - 1)$$

$$= 5n - 7G$$

The formula for the relationship between the T-total, T-number and grid size is 5n – (7G) (G is the grid size).

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

T-number	T-total
16	31
24	127
23	164

Trying to find the formula like the questions before:

1	2	3
	9	
	16	

		19
24	25	26
		33

	23	
	30	
36	37	38

The T-number as n, and the T-number missing all the other numbers in the T-shape but itself.

0°

n – 1	n – 2	n – 3
	n – 9	
	n	

90°

		n – 19
n	n – 25	n – 26
		n – 33

180°

	n	
	n – 30	
n – 36	n – 37	n – 38

Checking my formulas,

90°: $5n + 7 = 5(33) + 7 = 165 + 7 = 172$

T-total of T-shape with T-number of 33; $33 + 34 + 35 + 28 + 42 = 172$.

This formula works for a 7 by 7 grid.

180°: $5n + 35 = 5(32) + 35 = 165 + 35 = 195$

T-total of T-shape with T-number of 32; $32 + 39 + 45 + 46 + 47 = 209$.

This formula doesn't work.

$209 - 195 = 14$
$35 + 14 = 49$
$5n + 49$ should be the formula. Checking the formula,

$5n + 49 = 5(23) + 49 + 164$

T-total of T-shape with T-number of 23; $23 + 30 + 36 + 37 + 38 = 164$.

This formula works.

270°

16		
23	24	25
30		

25 – 16		
25 – 23	25 – 24	25
28 – 30		

using n instead of T-number.

$$n + [n - (25 - 24)] + [n - (25 - 23)] + [n - (25 - 16)] + [n - (25 - 30)]$$
$$= n + (n - 1) + (n + 2) + (n + 9) + (n - 5)$$
$$= n + n - 1 + n + 2 + n + 9 + n - 5$$
$$= 5n + 5$$

Checking my formula,

$5n + 5 = 5(25) + 5 = 125 + 5 = 130$

T-total of T-shape with T-number of 25; $28 + 24 + 23 + 16 + 30 = 118$.

This formula doesn't work.

$130 - 118 = 12$
$5 - 12 = -7$

The formula is $5n - 7$.

Strand 2 Mark 6

The candidate has obtained many expressions for the T-totals of various sizes of grids and these have been correctly expressed and justified by the adding together of the relevant terms $n + (n - 10) + (n - 11)$, etc. The candidate has then gone on to obtain the overall expression $5n - 7g$ as the total for any size of grid. All variables have been defined and the work warrants a mark of 6 in the second strand.

Strand 3 Mark 6

The work in the next section is taken from the task 'Beyond Pythagoras'.

Table with listed Pythagorean Triads, Perimeters and Areas:

I have decided to list 15 set of values in the table. The instructions have restricted the edge 'a' column to consecutive odd integers starting from three. Also, both the other edges 'b' and 'c' have to be positive integers with a difference of 1 between them. I have complete the rest of the table by studying the given numbers and then continuing the table step by step with the formulas I find. I shall explain how I completed the table afterwards and list what patterns or formulas I find.

Triad no.	edge 'a'	edge 'b'	edge 'c'	Perimeter	Area
1	3	4	5	12	6
2	5	12	13	30	30
3	7	24	25	56	84
4	9	40	41	90	180
5	11	60	61	132	330
6	13	84	85	182	546
7	15	112	113	240	840
8	17	144	145	306	1224
9	19	180	181	380	1710
10	21	220	221	462	2310
11	23	264	265	552	3036
12	25	312	313	650	3900
13	27	364	365	756	4914
14	29	420	421	870	6090
15	31	480	481	992	7440

I completed the table by continuing the edge 'a' column and then I found out that column 'b' increased by consecutive multiples of the four times table starting at eight. I saw that edge 'c' was one more than edge 'b' and I got the perimeter by adding all the edges up. Lastly I calculated the areas by using the formula of half the base times the perpendicular height. I then checked if the values were correct using Pythagoras' theorem.

From the table I can see the following patterns:

- Edge 'b' is always even
- Edge 'c' is always odd
- The perimeter is always even
- The area is always even

• Units in the edge 'b' column read: 4, 2, 4, 0, 0, 4, 2, 4, 0, 0...
• Units in the edge 'c' column read: 5, 3, 5, 1, 1, 5, 3, 5, 1, 1...
• Edge 'b' increases by consecutive multiples of 4, starting from 8 I
• Units in perimeter column go: 2, 0, 6, 0, 2, 2, 0, 6, 0, 2...
• Units in area column go: 6, 0, 4, 0, 0, 6, 0, 4, 0, 0...
• Perimeter increases by consecutive multiples of 8 starting from 18

From the table I have found the following general formulas:

$\frac{a^2 - 1}{2} = b$

$\frac{b}{4} =$ Triangular no.

(Triad no. x 2) + 1 = a

b + 1 = c 1

$\frac{ab}{2}$

$\frac{ab}{\text{Triad no.}} =$ Perimeter

I will now rearrange the above equations and substitute the triad no. for 'n' and have the equations in terms of 'n' so I can find the which follow the guidelines set at the beginning.

- 2n + 1 = a
- $\frac{a^2 - 1}{2} = b$

the $a^2 = (2n + 1)^2$

$= (2n + 1)(2n + 1)$

$= 4n^2 + 2n + 2n + 1$

$= 4n^2 + 4n + 1$

So now it is $\frac{4n^2 + 4n + 1 - 1}{2}$

$\frac{4(n^2 + n)}{2} = b$

$2(n^2 + n) = b$

Another way that I could of found the formula to 'b' in terms of 'n' was to have used the formula:

$\frac{b}{4} =$ Triangular number

values of 'b': $\frac{4}{4}, \frac{12}{4}, \frac{24}{4}, \frac{40}{4}, \frac{60}{4}, \frac{84}{4}, \frac{112}{4}, \frac{144}{4}$

= 1, 3, 6, 10, 15, 21, 28, 36

These are all consecutive triangular numbers.

The formula for the 'n' term of a triangle number is $\frac{n^2+n}{2}$

so: $\frac{b}{4} = \frac{n^2+n}{2}$

and: b = $\frac{n^2+n}{2} \times 4$

b= $\frac{4n^2+4n}{2}$

b = $2n^2+2n$

This is the same as above

- b + 1 = c
  the b = $2(n^2 + n)$

So now it is

- ab
  2 = Area
  the a = 2n + 1
  the b = $2(n^2 + n)$

So now it is $\frac{(2n+1)(2(n^2+2n)}{2}$ = Area

$\frac{4n^3+4n^2+2n^2+2n}{2}$ = Area

$\frac{4n^3+6n^2+2n}{2}$ = Area

$2n^3 + 3n^2 + n$ = Area

- ab
  n = perimeter
  the a = 2n + 1
  the b = $2(n^2 + n)$

So now it is $\frac{4n^3+6n^2+2n}{n}$ (taken from above) = Perimeter

$4n^2 + 6n + 2 = \text{Perimeter}$

So m terms of 'n' I have:

- $a = 2n+1$
- $b = 2n(n + 1)$
- $c = 2n(n + 1) + 1$
- Area $= 2n^3 + 3n^2 + n$
- Perimeter $= 4n^2 + 6n + 2$

I can check these formulas are correct by substituting them into Pythagoras' theorem. Pythagoras' theorem states that $a^2 + b^2 = c^2$. I have found 'a' as
$2n + 1$, 'b' as $2(n^2 + n)$ and 'c' as $2(n^2 + n) + 1$.

So $(2n + 1)^2 + [2(n^2 + n)]^2 = [2(n^2 + n) + 1]^2$

$(2n + 1)^2 + (2n^2+2n)^2 = (2n^2 + 2n + 1)$

$(2n +1)(2n +1) + (2n^2 + 2n)(2n^2 + 2n) = (2n^2 + 2n + 1)(2n^2 + 2n + 1)$

I

$4n^2 + 4n + 1 + 4n^4 + 4n^3 + 4n^3 + 4n^2 = 4n^4 + 4n^3 + 2n^2 + 4n^3 + 4n^2 + 2n + 2n^2 + 2n+1$

$$4n^4 + 8n^3 + 8n^2 + 4n + 1 = 4n^4 + 8n^3 + 8n^2 + 4n + 1$$

These values equate which means my equations are correct to the requirements of Pythagoras' theorem and can be used to find any Pythagorean triad when the triad number is given.

I can also check that my formulas for perimeter and area work: I know that the perimeter is the length round the whole shape and so it is the sum of the edges: $a + b + c$.

So: $(2n + 1)+2n(n + 1) + 2n(n + 1) + 1 = 4n^2 + 6n + 2$

$2n + 1 + 2n^2 + 2n + 2n^2 + 2n + 1 = 4n^2 + 6n + 2$

$4n^2 + 6n + 2 = 4n^2 + 6n + 2$

With area, I know that area $= \frac{1}{2} \times a \times b$

So: $\frac{1}{2} (2n + 1)\, 2n(n + 1) = 2n^3 + 3n^2 + n$

$\frac{1}{2}(2n + 1)(2n^2 + 2n) = 2n^3 + 3n^2 + n$

$\frac{1}{2}(4n^3 + 4n^2 + 2n^2 + 2n) = 2n^3 + 3n^2 + n$

$2n^3 + 3n^2 + n = 2n^3 + 3n^2 + n$

These both sets of values equate so the formulas are correct.

With these formulas I can find all the other values when given the triad number. If the given value is the area, perimeter, or any edge I can find all the other values by rearranging the equations accordingly. So if I am told the area I can subsequently find out the perimeter, the Triad number and all of the sides of the tight angled triangle. Now I can apply these formulas to situations involving the above listed right ang1ed triangles and others that follow the same guidelines.

example: If I take Triad no. 7 I can find all the other values just by knowing the triad number.

step 1:	use the formula to find edge 'a'	$2n + 1 = a$
		$(2 \times 7) + 1 = a$
		$14 + 1 = a$
		$15 = a$
		Now I have 'a'
step 2:	use the formula to find edge 'b'	$2(n^2 + n) = b$
		$2(7^2 + 7) = b$
		$2(49 + 7) = b$
		$2(56) = b$
		$112 = b$
		Now I have 'b'
step 3:	use the formula to find edge 'c'	$2(n^2 + n) + 1 = c$
		$2(7^2 + 7) + 1= c$
		$2(49 + 7) + 1= c$
		$2(56) + 1= c$
		$112 + 1 = c$
		$113 = c$
		Now I have 'c'
step 4:	use the formula to find the perimeter	$4n^2 + 6n + 2$ perimeter
		$4(7)^2 + 6(7) + 2 =$ perimeter
		$4(49) + 42 + 2 =$ perimeter
		$196 + 44 =$ perimeter
		24O perimeter
		Now I have the perimeter

step 5: use the formula to find the area

$2n^3 + 3n^2 + n = \text{area}$
$2(7)^3 + 3(7)^2 + 7 = \text{area}$
$2(343) + 3(49) + 7 = \text{area}$
$686 + 147 + 7 = \text{area}$
$840 = \text{area}$
Now I have the area

I have all the values and now I can check them with my table.

	a	b	c	perimeter	area
my values	15	112	113	240	840
table's values	15	112	113	240	840

They are the same.

Extension:

What if I change the boundaries..?

- I make 'b' the same as 'a', thus making the triangle Isosceles as well as right angled.
- I make 'a' an even number.
- I make 'b' and 'c' more than one number apart

Experiment to see if my formulas still work in these circumstances:

Making 'b' equal to 'a':

Using my formula: $b + 1 = c$
$7 + 1 = c$
$8 = c$

Using Pythagoras to check: $a^2 + b^2 = c^2$
$7^2 + 7^2 = c^2$
$49 + 49 = c^2$
$98 = c^2$
$\sqrt{98} = c$
$9.899494937 = c$

These values do not equate. Pythagoras' theorem is definitely correct and so mine is incorrect in this situation so I will try again:

Using my formula : $b + 1 = c$
$5 + 1 = c$
$6 = c$
$a = 5$

Using Pythagoras to check: $a^2 + b^2 = c^2$
$52 + 52 = c^2$
$25 + 25 = c^2$
$50 = c^2$
$\sqrt{50}$
$7.0710678\ 12 = c$

These values do not equate either.

So, my values do not match the values I get from using Pythagoras' theorem~ this means that this formula I have found does not work in these circumstances. This also means that when the right-angled triangle is isosceles as well, then the values will not obey the rules set at the beginning. This also shows that this value cannot be added into the table because it is not an integer value.

Making 'a' an even number:

make $a = 4$

Now use my formula; $\frac{a^2 - 1}{2} = b$ to find 'b',

(I must use this because I need two sides to apply Pythagoras. So the 'a' and 'b' values are now proportionate to the ones at the beginning in the table.)

$$\frac{4^2 - 1}{2} = b$$

$$\frac{16 - 1}{2} = b$$

$$\frac{15}{2} = b$$

$7.5 = b$

'b' is not an integer value.

Now using Pythagoras' theorem $a^2 + b^2 = c^2$ I can find value 'c':

$4^2 + 7.5^2 = c^2$
$16 + 56.25 = c^2$
$72.25 = c^2$
$\sqrt{72.25} = c$
$8.5 = c$

This time as well I do not have an integer value but what drew my attention was the fact that b' and 'c' are still only one apart, this would also of happened if I had used my equation of 'b + 1'. I shall repeat this experiment to check if this was luck, but I shall change 'a' to 6.

$$\frac{6^2 - 1}{2} = b$$

$$\frac{36 - 1}{2} = b$$

$$\frac{35}{2} = b$$

$17.5 = b$

Now I shall use Pythagoras' theorem to find out if 'c' is still one more than 'b'

$a^2 + b^2 = c^2$
$6^2 + 17.5^2 = c^2$
$36 + 306.25 = c^2$
$342.25 = c^2$
$\sqrt{342.25} = c^2$
$18.5 = c$

This formula still works when ~a'is an even number as †b' and 'c' are still always one apart. I have also noticed that 'b' and 'c' have ended in '.5' which is halfway. I shall no draw up a small table progressing from all integer values for 'a' starting from 3 and finding 'b' by the formula $\frac{a^2 - 1}{2} = b$.

Triad no.	edge 'a'	edge 'b'	edge 'c'	Perimeter	Area
1	3	4	5	12	6
1.5	4	7.5	8.5	20	15
2	5	12	13	30	30
2.5	6	17.5	18.5	42	52.5
3	7	24	25	56	84
3.5	8	31.5	32.5	72	126
4	9	40	41	90	180
4.5	10	49.5	50.5	110	247.5
5	11	60	61	132	330

From this table I spotted that if the half values are rounded up to integer values then they would be the mid-point between the values either side of it this occurs for both the b' and 'c' values. For example, with triad number 4, if the 'b' value 17.5, was rounded up to 18 that would be the mid-point value between 12 and 24. This is because there is a difference of 12 and half of twelve is 6 and when 6 is added to 12 the answer is 18.
The most interesting thing I found was that all my formulas in terms of 'n' work here when I have made 'n' halfway values. I did this because I did not want to disrupt the pattern and this shows that the pattern still works for odd integers of 'a'.

I have not randomly chosen 'b' as any number I have restricted it to be linked to 'a' by the formula: $\frac{a^2 - 1}{2} = b$.
This means that when 'b' is restricted that the formulas will still work but when it is not the formulas might not work. So now I will make 'a' an even number but randomly choose 'b'.
Let a = 10,and b = 29 and use Pythagoras' theorem to find 'c'.

$10^2 + 29^2 = c^2$
$100 + 841 = c^2$
$941=c^2$
$\sqrt{941}=c^2$
$30.6757233 = c$

So when edge 'a' is an even number a formula is needed to link 'a' and 'b' and then all the formulas will continue to work as before but without the link nothing will work. There are non integer values in the table but they are only halves and still fit the pattern. However, when edge 'a' is an even number I can still make triangles with integer values for all edges. This can be done, but I must break the rules and patterns and form new ones from a new table I shall form.

I have found from trial and error that if I make edge 'a' equal 6, then edge 'b' will be 8 and edge 'c' will be 10. I got this by choosing 'a' as 6 and then choosing the integer value up from 'a' for 'b' and calculating 'c' from there. If 'c' did not turn out as an integer value then I increased 'b' till it worked. I went on, by making 'a' go up to the next even integer value - 8, from there, using the above method of trial and error, I found that 'b' was 15 and 'c' was 17. Then I made 'a' equal 10, I found that 'b' was 24 and 'c' was 26. Now I have enough values to find some patterns such as 'c' is 2 more than 'b' and the difference between the 'b' values increases by consecutive odd numbers. I can find the perimeter by adding the edges and I can find the area by using the formula of a half multiplied by 'a' multiplied by 'b'. This is enough information to enable me to complete a substantial table.

n	edge 'a'	edge 'b'	edge 'c'	perimeter	area
1	6	8	10	24	24
2	8	15	17	40	60
3	10	24	26	60	120
4	12	35	37	84	210
5	14	48	50	112	336
6	16	63	65	144	504
7	18	80	82	180	720
8	20	99	101	220	990
9	22	120	122	264	1320
12	28	195	197	420	2730
13	30	224	226	480	3360
14	32	255	257	544	4080
15	34	288	290	612	4896

Strand 3 Mark 6

The candidate has found symbolic expressions for the cases when the smallest side is odd and the other two differ by one. All the results are justified. The candidate then decides to consider the cases where the smallest side is even. This is moving the task into the analysis stage. For the logic and justification used so far, a mark of 6 is awarded in the third strand.

Another example of **Strand 3 Mark 6** is taken from the ‘Open Box Problem’ below. The candidate has slightly modified the task to consider containers rather than a simple cuboid-type box.

The maximum value possible for x in a piece of card 20 cm x 20 cm falls between 3 cm and 4 cm.

x = 3.5	x = 3.3	x = 3.4
$(20 - 2x)^2x$	$(20 - 2x)^2x$	$(20 - 2x)^2x$
$(20 - 7)^2 \times 3.5$	$(20 - 6.6)^2 \times 3.3$	$(20 - 6.8)^2 \times 3.4$
169 x 3.5 = 591.5	179.56 x 3.3 = 592.548	174.24 x 3.4 = 592.416

The maximum value possible for x falls between 3.3 cm and 3.4 cm.

x = 3.35	x = 3.34	x = 3.33
$(20 - 2x)^2x$	$(20 - 2x)^2x$	$(20 - 2x)^2x$
$(20 - 6.7)^2 \times 3.35$	$(20 - 6.68)^2 \times 3.34$	$(20 - 6.66)^2 \times 3.33$
176.89 x 3.35 = 592.5815	177.4224 x 3.34 = 592.548	177.9556 x 3.33 = 592.592148

x = 3.32
$(20 - 2x)^2x$
$(20 - 6.64)^2 \times 3.32$
178.4896 x 3.32 = 592.585472

In a piece of card of 20 cm x 20 cm, the maximum size of x which will give the maximum value possible is 3.33 cm. I will now differentiate my answer to check that it is correct.

Use differentiation:

$$
\begin{aligned}
y = \quad & (20 - 2x)^2x \\
& (20 - 2x)(20 - 2x)x \\
& (400 + 4x^2 - 40x - 40x)x \\
& 4x^3 - 80x^2 + 400x \\
& 4(3)x^2 - 80(2)x^1 + 400(1)x^0 \\
& 12x^2 - 160x + 400 = 0
\end{aligned}
$$

Use factorisation:

$$
\begin{aligned}
& 4(3x^2 - 40x + 100) \\
& 4(3x^2 - 30x - 10x + 100) \\
& 4((3x - 10)(x - 10)) = 0
\end{aligned}
$$

$3x - 10 = 0 \qquad x - 10 = 0$

$3x = 10 \qquad x = 10$ – this isn't possible as the card would be folded in half

$x = 3.33$

I will now differentiate the quadratic equation, substituting x with the value I calculated for it.

$12x^2 - 160x + 400 = 0$

$12(2)x^1 - 160(1) + 400(0)$

$24x - 160 =$

$24(3.33) - 160$

$79.92 - 160 = -80.08$

The answer is negative and therefore my answer for the value of the volume is a maximum for this value of x.

The best size for x on a piece of card 20 cm x 20 cm is 3.33 cm.

I will now work out the best size for x on a piece of card of size 12 cm x 12 cm, although I will work it our first using differentiation and then I will use the trial and improvement method to check it.

Size of card = 12 cm x 12 cm. $0 < x < 6$

Use differentiation:

$y = (12 - 2x)^2x$

$(12 - 2x)(12 - 2x)x$

$4x^3 - 48x^2 + 144x$

$4(3)x^2 - 48(2)x^1 + 144(1)x^0$

$12x^2 - 96x + 144 = 0$

Use factorisation:

$4(3x^2 - 24x + 36)$

$4(3x^2 - 18x - 6x + 36)$

$4((3x - 6)(x - 6)) = 0$

$3x - 6 = 0 \qquad x - 6 = 0$

$3x = 6 \qquad x = 6$ – this isn't possible as the card would be folded in half

$x = 2$

I will now differentiate the quadratic equation, substituting x with the appropriate value.

$12x^2 - 96x + 144 = 0$
$12(2)x^1 - 96(1)x^0 + 144(0)$
$24x - 96 =$
$24(2) - 96$
$48 - 96 = -48$

The answer is negative and therefore my answer for the value of the volume is a maximum for this value of x.

I will now use the trial and improvement method to check my answer.

Size of card = 12 cm x 12 cm. $0 < x < 6$

x = 2	x = 1	x = 1.99
$(12 - 2x)^2x$	$(12 - 2x)^2x$	$(12 - 2x)^2x$
$(12 - 4)^2 \times 2$	$(12 - 2)^2 \times 1$	$(12 - 3.98)^2 \times 1.99$
64 x 2 = 128	100 x 1 = 100	64.3204 x 1.99 = 127.997596

x = 3	x = 2.1
$(12 - 2x)^2x$	$(12 - 2x)^2x$
$(12 - 6)^2 \times 3$	$(12 - 4.2)^2 \times 2.1$
36 x 3 = 108	60.84 x 2.1 = 127.764

The best possible size for x on a piece of card of size 12 cm x 12 cm is 2 cm.

I am now going to attempt to find a pattern between the length of the square card and the length of square x which will give the greatest volume possible.

Length of card	Best size for x
12 cm	2 $(\frac{6}{3})$
16 cm	2.67 $(\frac{8}{3})$
20 cm	3.33 $(\frac{10}{3})$

I have noticed that the best size of x can be worked out when you divide the original length by 6.

Therefore a formula for quickly finding the size of x on a square piece of card would be written as $\frac{L}{6}$, where L = the length of the side. (As it is a square, which side is divided by six doesn't matter.) I predict, using this formula ($\frac{L}{6}$), that a piece of card of size 30 cm x 30 cm would be best suited to the size of x being 5 cm ($30 \div 6 = 5$) to avail of the maximum volume possible.

I will now test my prediction using differentiation.

$y = (30 - 2x)^2x$
$(30 - 2x)(30 - 2x)x$
$(900 + 4x^2 - 60x - 60x)x$
$4x^3 - 120x^2 + 900x$
$4(3)x^2 - 120(2)x^1 + 900(1)x^0$
$12x^2 - 240x + 900 = 0$

Use factorisation:

$4(3x^2 - 60x + 225)$
$4(3x^2 - 15x - 45x + 225)$
$4((3x - 15)(x - 15)) = 0$
$3x - 15 = 0 \qquad x - 15 = 0$
$3x = 15 \qquad x = 15$ – this isn't possible as the card would be folded in half
$x = 5$

$12x^2 - 240x + 900 = 0$
$12(2)x^1 - 240(1) + 900(0)$
$24x - 240 =$
$24(5) - 240$
$120 - 240 = -120$

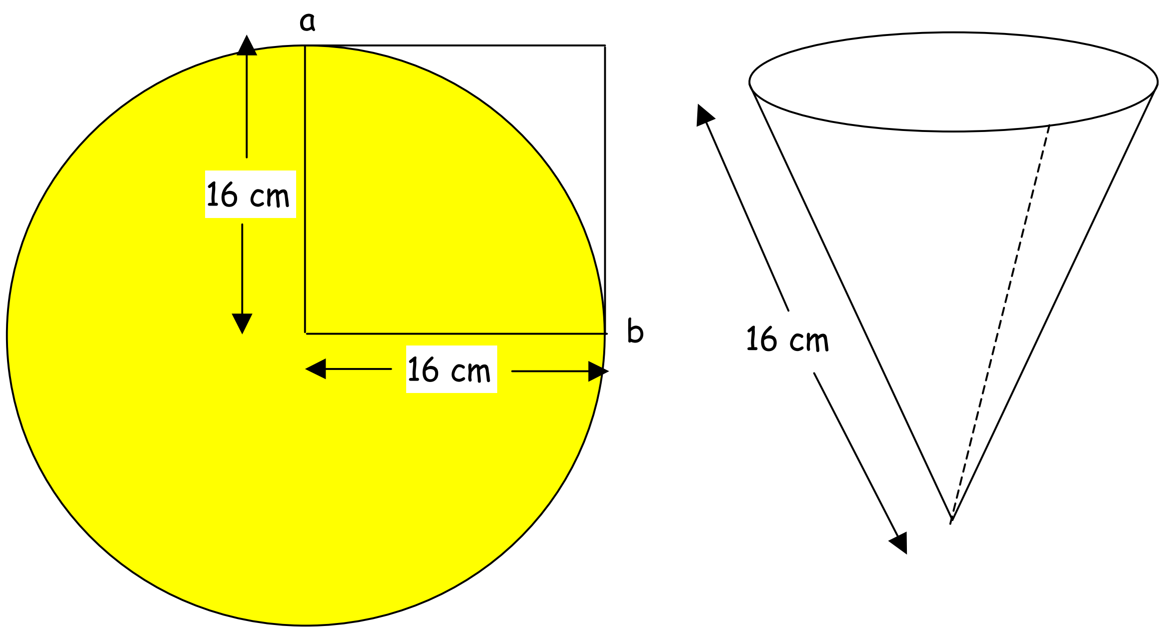
The answer is negative and therefore my answer for the value of the volume is a maximum for this value of x.

For a piece of card of size 30 cm x 30 cm, the best size for x which would give the maximum value possible is 5 cm. This shows that my formula is correct for any square piece of card. I will now consider possible extensions concerning such factors as different shapes, e.g. a cone, a cylinder or a rectangle. I will also see which shape gives the maximum volume possible, when cut from similar pieces of card of the same size, e.g. from a piece of card of size 16 cm x 16 cm, and so would be best suited for producing boxes to meet the requirement for the specified task.

Formula for any container derived from a square piece of card, cuboid in shape = $\frac{L}{6}$, where L = length of one side of card without any sections removed.

Extension 1: Cones

After having investigated the possibility of making the strawberry box from a piece of square card, producing a cuboid as a container, I am now going to investigate the possibility of making the container from another shape cut from a square piece of card. In this case the shape will be a cone. It will be cut from a square piece of card of size 16 cm x 16 cm, and I will attempt to calculate the volume of the finished container.



When cut along the curved line ab, there will be a piece of card which will be a quarter of the circle. (The non-shaded area is waste). The card will then be folded so that point a is joined up with point b, forming the base of the cone. The cone will have a slant height of 16 cm. To calculate the volume I will follow a number of steps. First I will work out the circumference of the base. To do this I will pretend that the curve ab is a quarter of a circle, of which 16 cm is the radius. I will then find the circumference of this circle and then divide it by 4 to get the length of the curve ab. Then, using this length, I will work my way back all the formula which is used to find the circumference of a circle, which is $2\pi r$, with the aim of discovering the radius of the base of the cone. Then, using Pythagoras' Theorem, I will attempt to calculate the height of the cone.

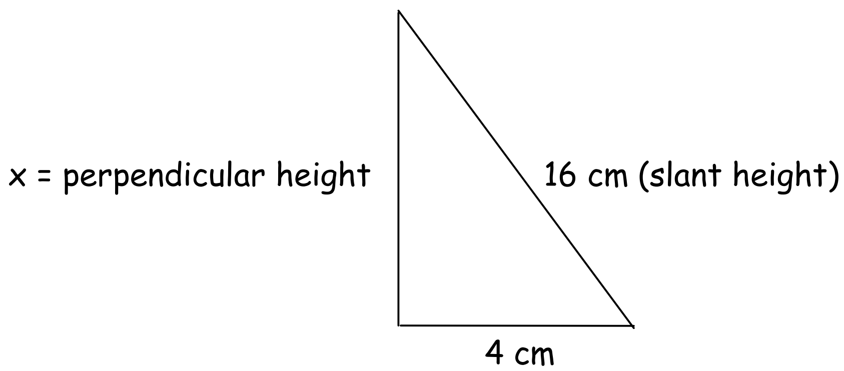
Then I will fit these numbers into the formula used to find the volume of a cone, which = $\frac{1}{3}\pi r^2 h$.

$C = 2\pi r$ — C = circumference
$C = 2\pi(16)$ — r = radius = 16 cm
$C = 32\pi$
$C = 100.53$ cm
$\frac{1}{4}C = \dfrac{100.53}{4} = 25.1327$ cm

Circumference of base = 25.1325 cm. Now I will find the radius of the base.

$C = 2\pi r$
$25.1327 = 2\pi r$
$\dfrac{25.1327}{2\pi} = r$
$\dfrac{25.1327}{2\pi} = 4$
$r = 4$

The radius of the base of the cone = 4 cm. I will now use Pythagoras' Theorem to calculate the height of the cone.



16 cm = length of slant height of cone
4 cm = length of radius of base of cone
x cm = length of perpendicular height of cone.
$4^2 + x^2 = 16^2$
$16 + x^2 = 256$
$x^2 = 256 - 16$
$x^2 = 240$
$\therefore x = \sqrt{240} = 15.49$ cm

The height of the cone = 15.49 cm.

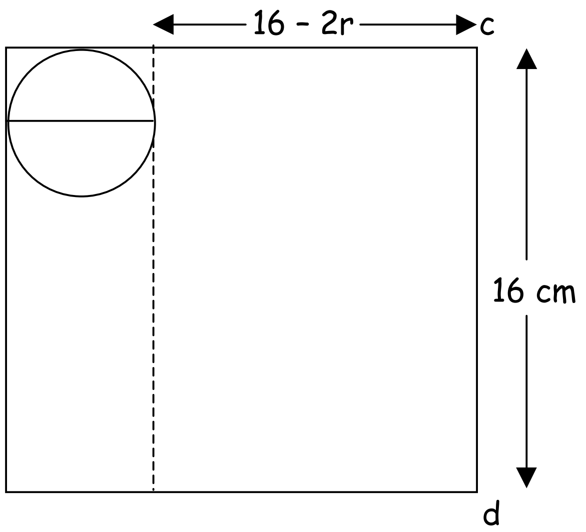
I will now use the numbers to calculate the volume of the cone. This volume will be the maximum volume possible achievable on a cone cut from a piece of card of size 16 cm x 16 cm.

$$\begin{aligned} V &= \tfrac{1}{3}\pi r^2 h \\ &= \tfrac{1}{3}\pi 4^2 \times 15.49 \\ &= \tfrac{1}{3}\pi \times 16 \times 15.49 \\ &= 1.047 \times 16 \times 15.49 \\ &= 259.537441 \text{ cm} \end{aligned}$$

The volume of the cone was 259.537 cm (3 d.p.). This is the best possible volume for a cone from a piece of card 16 cm x 16 cm.

Extension 2: Cylinders

Having investigated using a cone as a container. I am now going to investigate the possibility of using a cylinder as the container for the desired application. Again, the cylinder will be cut from a piece of card of size 16 cm x 16 cm, and the method employed will give the maximum volume possible for a cylinder cut from a piece of card of size 16 cm x 16 cm.



I will employ two methods to determine the maximum volume possible for a cylinder cut from a piece of card of size 16 cm x 16 cm. In Method 1, the side cd (16 cm) will be wrapped in a circle, point c, joined to point d. The circumference of the base will be 16 cm. From here I will work back along the formula $2\pi r$ to determine what r is equal to. Then I will incorporate the size of r into the volume formula, which is $V = \pi r^2 h$. In Method 2, points b and c will be joined to form the base, with a circumference of 16 – 2r. I will then determine what r is equal to this time and once I have done so I will incorporate r into the volume formula and then I will see which way of designing the cylinder gives the maximum volume possible on a piece of card of size 16 cm x 16 cm.

Method 1:

$C = 2\pi r$

$16 = 2\pi r$

$r = \frac{16}{2\pi}$

$r = \frac{16}{6.28} = 2.548$

The radius is equal to 2.548 cm (3 d.p.). I will now calculate the volume:

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi(2.548)^2 \times (16 - 2r) \\ &= 3.14 \times 6.492304 \times (16 - 5.096) \\ &= 3.14 \times 6.492304 \times (10.904) \\ &= 222.28714 \text{ cm} \end{aligned}$$

The volume of the cylinder when the height equals 16 – 2r is 222.28714 cm. This is the maximum volume possible for this method. I know this because all of the card cut will be used and there will be no overlap.

Method 2:

$C = 2\pi r$

$16 - 2r = 2\pi r$

$16 = 2\pi r + 2r$

$8 = \pi r + r$

$8 = r(\pi + 1)$

$16 = 2r(\pi + 1)$

$r = \frac{16}{2(\pi + 1)}$

$r = 1.931624056$

The radius is equal to 1.931624056 cm. I will now incorporate this into the volume formula $V = \pi r^2 h$.

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi(1.932)^2 \times 16 \\ &= 3.14 \times 3.732624 \times 16 \\ &= 187.5270298 \text{ cm} \end{aligned}$$

The volume of the cylinder when the height equals 16 is 187.5270298 cm. This is the maximum volume possible for this method.

I have found that a larger volume was obtained when the height equalled 16 cm. Method 1, therefore, is the best method with which to design a cylindrical container.

Conclusion:

I have compared these results with the results of the original investigation and I have found that a square would be the best shape over a cylinder and a cone.

Max volume:
Square = 303.407052 cm^3
Cone = 259.537411 cm^3
Cylinder = 222.28714 cm^3

This is because the square is the most economical shape as it removes the least space from its surface area as waste, whilst the other shapes remove relatively a lot more, leaving less card with which to obtain a large volume. Therefore a square would be the best shape from which to design a box to fit the problem stated in the original investigation.

Strand 3 Mark 6

The candidate generalises for the square-based cuboids. The candidate then clearly explains the method and solution for a particular cone and cylinder. Each of these gives evidence of the candidate's use of reasoning and logic in making further progress in the task. The candidate could have moved to analyse the situation more generally but did not do so. The work, however, is worthy of mark 6 in strand 3.

MARK 7

The work in this section is taken from the task "Triminoes".

Number of cards in a set when the only numbers used are 0 and 1:

0	0	0	1
0	0	1	1
0	1	1	1

= 4 possibilities

Number of cards in a set when the only numbers used are 0, 1 and 2:

0	0	0	0	0	0	1	1	1	2
0	0	0	1	2	1	1	1	2	2
0	1	2	1	2	2	1	2	2	2

= 10 possibilities

Number of cards in a set when the only numbers used are 0, 1, 2 and 3:

0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	2	2	2	3
0	0	0	0	1	2	3	1	2	1	1	1	2	3	2	2	2	3	3
0	1	2	3	1	2	3	2	3	1	2	3	2	3	3	2	3	3	3

= 20 possibilities

Depending on the number of integers used, you can find how many cards there will be in a set. The number of cards in a set is found by adding triangle numbers. For example if there are two integers used you add the first two triangle numbers.

e.g. 0, 1: $1 + 3 = 4$
0, 1, 2: $1 + 3 + 6 = 10$
0, 1, 2, 3: $1 + 3 + 6 + 10 = 20$

when the largest number on the cards is 4, then the number of cards in the set will be $1 + 3 + 6 + 10 + 15 = 35$.

Test:

(15)

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	2	3	4	1	1	1	2	2	3
0	1	2	3	4	1	2	3	4	2	3	4	3	4	4

= 35 possibilities

				(10)									(6)				(3)		(1)
1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	3	3	3	4
1	1	1	1	2	3	4	2	2	3	2	2	2	3	4	3	3	3	4	4
1	2	3	4	2	3	4	3	4	4	2	3	4	3	4	4	3	4	4	4

The number of cards in a set is found by adding triangle numbers in order. Above shows the different types of cards which you can have when the numbers used are 0, 1, 2, 3 and 4. The number in brackets show the number of cards in each block. They are all triangle numbers, this is why the amount of cards in the set is found by adding triangle numbers.

Number of integers (n)	Largest number on cards (n – 1)	No of cards in set
1	0	1
2	1	4
3	2	10
4	3	20
5	4	35

To find the number of cards in a set when the number of integers is known:

$$\sum_{i=1}^{i=n} \frac{i(i+1)}{2}, \quad \text{e.g. sum of triangle numbers}$$

$$= \sum_{i=1}^{i=n} \frac{i^2+i}{2} \quad = \frac{1}{2}\left(\sum_{i=1}^{i=n} i^2 + \sum_{i=1}^{i=n} i\right)$$

Formula for $\sum_{i=1}^{i=n} i^2$, e.g. sum of square numbers. Using the method of differences:

1
} 4
5 } 5
} 9 } 2
14 } 7 } 0
} 16 } 2
30 } 9
} 25
55

$ax^3 + bx^2 + cx + d$

$a + b + c + d = 1$
$8a + 4b + 2c + d = 5$
$27a + 9b + 3c + d = 14$
$64a + 16b + 4c + d = 30$

$8a + 4b + 2c + d = 5$
$a + b + c + d = 1$ } $7a + 3b + c = 4$

$27a + 9b + 3c + d = 14$
$8a + 4b + 2c + d = 5$ } $19a + 5b + c = 9$

$64a + 16b + 4c + d = 30$
$27a + 9b + 3c + d = 14$ } $37a + 7b + c = 16$

$19a + 5b + c = 9$
$7a + 3b + c = 4$ } $12a + 2b = 5$

$37a + 7b + c = 16$
$19a + 5b + c = 9$ } $18a + 2b = 7$

$18a + 2b = 7$
$12a + 2b = 5$ } $6a = 2$

$a = \frac{1}{3}$

Substitute a back into the formula 12a + 2b = 5 to find b:

$b = \frac{5 - 12(\frac{1}{3})}{2} = \frac{1}{2}$

Substitute b and a back into the formula to find c:

$37a + 7b + c = 16, \quad c = 16 - 37(\frac{1}{3}) - 7(\frac{1}{2}) = \frac{1}{6}$

Substitute a, b and c back into the formula to find d:

$a + b + c + d = 1, \quad d = 1 - \frac{1}{3} - \frac{1}{2} - \frac{1}{6} = 0$

Formula for sum of square numbers:

$$\tfrac{1}{3}n^3 + \tfrac{1}{2}n^2 + \tfrac{1}{6}n \;=\; n(\tfrac{1}{3}n^2 + \tfrac{1}{2}n + \tfrac{1}{6}) \;=\; \tfrac{1}{6}n(2n^2 + 3n + 1)$$

$$= \tfrac{1}{6}n(2n + 1)(n + 1)$$

$\sum_{i=1}^{i=n} i^2$ = 1 + 2 + 3 + . . . + n, e.g. triangle numbers.

Formula for triangle numbers = $\tfrac{1}{2}n(n - 1)$

Substitute $\tfrac{1}{6}n(2n + 1)(n + 1)$ and $\tfrac{1}{2}n(n - 1)$ back into the formula

$$\frac{1}{2}\left(\sum_{i=1}^{i=n} i^2 + \sum_{i=1}^{i=n} i\right)$$

$$= \frac{1}{2}\left[\frac{n}{6}(n+1)(2n+1) + \frac{1}{2}n(n+1)\right]$$

$$= \frac{n}{2}(n+1)\left[\frac{1}{6}(2n+1) + \frac{1}{2}\right]$$

$$= \frac{n}{2}(n+1)\left[\frac{(2n+1)}{6} + \frac{1}{2}\right]$$

$$= \frac{n}{2}(n+1)\left[\frac{(2n+1+3)}{6}\right]$$

$$= \frac{n(n+1)(n+2)}{6}$$

The formula $\frac{n(n+1)(n+2)}{6}$ tells you the number of cards in a set, when the number of integers is n.

e.g. number of integers = 1

$$\frac{1(1+1)(1+2)}{6} = \frac{1(2)(3)}{6} = \frac{6}{6} = 1$$

so when the number of integers is 1, the number of cards in the set is 1.

Supposing the largest number was known, e.g. 4, then (4 + 1) would be substituted in place of n as the largest number is one less than the number of digits.

e.g. largest number = 4

$$\frac{(4+1)[(4+1)+1][(4+1)+2]}{6} = \frac{5\times6\times7}{6} = 35,$$ number of cards in set.

Strand 1 Mark 7

The candidate has started to analyse the situation after many diagrams and tables of results (not included). Candidate looks for a way of finding the totals for any given number of integers. Realises that it is the sum of triangular numbers and develops an approach to find it, indicating a clear method.

Strand 2 Mark 7

Very good communication and a very convincing argument is put forward by the candidate. All algebra is correct.

Strand 3 Mark 7

The work is justified throughout by the approach used by the candidate. Candidate is very aware of the procedure and uses simultaneous equations in 4 unknowns to find the generalisation for square numbers and relates this back to find the sum of triangular numbers.

MARK 8

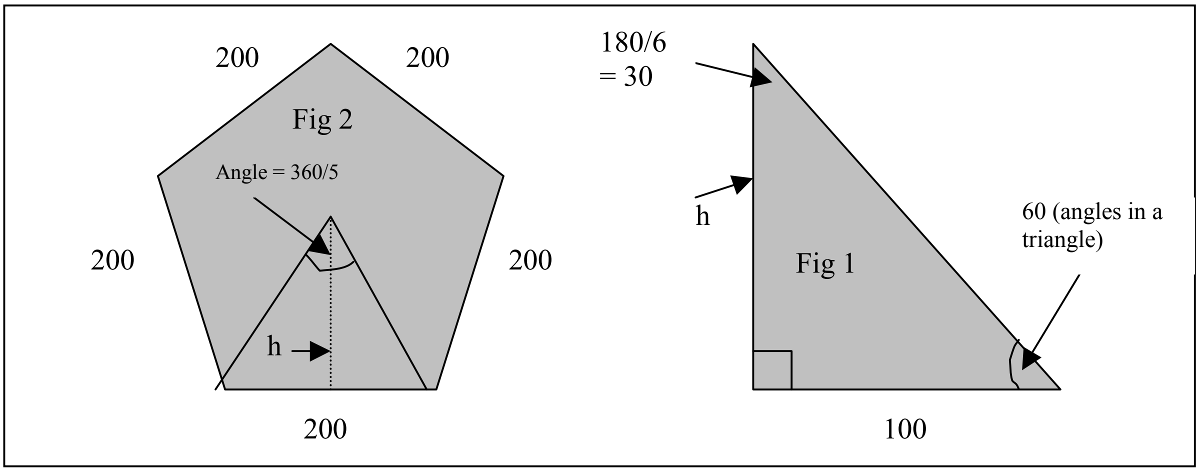
The work in this section is taken from the task 'The Fencing Problem'.

My prediction was correct; increasing the number of sides to six with the hexagon did increase the area. I now believe that I have investigated enough shapes to make a more general prediction. I predict that as the number of sides increases, so does the area of the shape.

From this prediction I can go about creating a formula that will give me the area of an 'n' sided polygon. That is, I can obtain a formula for the area of any regular polygon with any amount of sides.

To so this, I have to substitute the figures I am using in the equation into algebraic terms. I will assign the term 'n' to mean the number of sides that the polygon has.

Since I have to divide the polygon into triangles to work out its area, I will start translating this part of the equation into an algebraic term first.



To work out the area of the triangle, I have to use trigonometry to calculate the perpendicular height. This involves using individual angles and lengths. I will put these angles and lengths into algebraic terms.

The centre angle, as shown in figure 2, can be represented by **360/n** (n being the number of sides). Since I am working on half of the triangle, I will make this angle **180/n.** I don't actually need this angle in the equation, but it helps me put the remaining angle into algebra.

The remaining angle is as shown in figure 1, which, in this case is 60. I can calculate this angle as follows:

90 - 180/n = remaining angle

I will now put the lengths into algebraic terms. One side is equal to 1000/n. Because I need half of this to calculate the area, I will say that the side is equal to 500/n.

To calculate the area of the triangle, I need to use trigonometry to find the perpendicular height of the triangle.

If Tan = opposite/adjacent, then

Tan (90 – 180/n) = h / (500/n) so

Perpendicular height = Tan (90 - 180/n) x 500/n

I will add the term for half the base; then I will have a formula for working out the area of the triangle.

Area of full triangle = $\frac{1}{2}$ base x perpendicular height

Area = 500/n x tan (90 - 180/n) x 500/n

To calculate the formula for the area of the polygon, I need to multiply the area of the triangle by the number of sides that the polygon has (n).

Area of polygon = n x 500/n x tan (90 – 180/n) x 500/n

I can simplify this because I have many like terms. I have two 500/n terms, so this can become $500^2/n^2$. This leaves me with one 'n' term left to simplify. I can do this by altering as follows:

Area = 500^2/n x tan (90-180/n)

I now have a formula to calculate the area of any regular polygon. In order to see if the formula may be correct, I can test it.

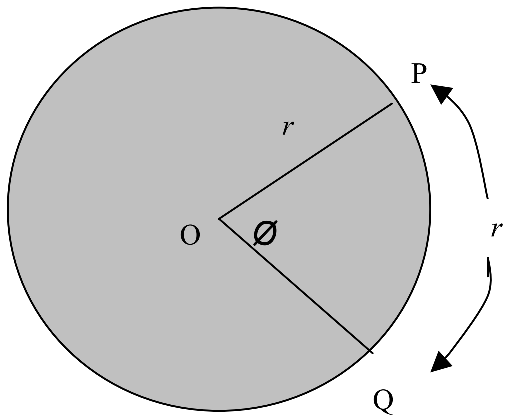
Area of pentagon using previous method (without rounding) = 68 819. 09602

Area using formula = (250 000/5) x tan (90-180/5)
= 50 000 x 1.37638192
= 68 819.09602

Even though this is only one test, I am completely satisfied that this formula will be correct for any polygon.

To further my investigation, I am going to compile a spreadsheet showing the areas of many polygons. I will adapt the formula I have just calculated into the spreadsheet format. To do this, I have had to convert the figures expressed in angles into **radians**.

One radian is the angle subtended at the centre by the arc of a circle whose length is equal to the radius of the circle.



In the diagram, the radius of the circle *r* and the length of the arc *PQ* is also *r*. As the arc is the same length as the radius of the circle, the angle Ø is 1 radian. This can be abbreviated to 1^c or 1 rad.

The angle made at the centre of the circle by an arc of length *r* is 1^c. The circumference of the circle is $2\pi r$, which is 2π times the length of this arc. So the angle of one revolution is $2\pi \times 1^c = 2\pi^c$ (two pi radians). One full turn is 360 degrees. One full turn in radians is 2π radians. Therefore:

$360 = 2\pi^c$

$180 = \pi^c$

$90 = \frac{\pi^c}{2}$

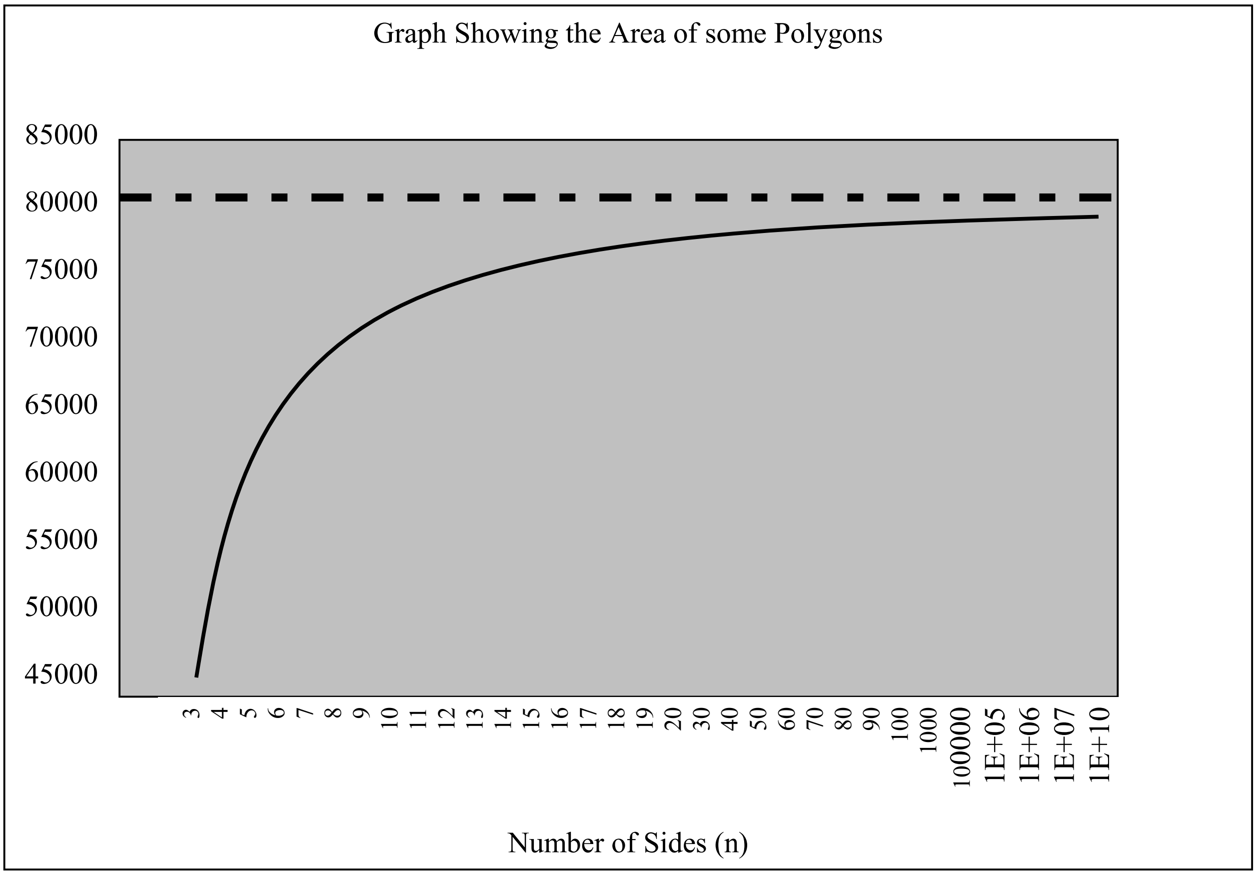
My formula is now in the revised format for spreadsheets:

Area = 250 000/n x tan (Pi/2 - Pi/n)

I will now separate the formulae into different parts so that I can enter this data into the different columns on the spreadsheet which follows:

	n(number of sides)	250000/n	=Tan (Pi/2-Pi/A1)	=B1 x C1
	A	B (1/2 base)	C (perpendicular height)	D (Area)
1	3	83333.33333	0.577350269	48112.52243
2	4	62500	1	62500
3	5	50000	1.37638192	68819.09602
4	6	41666.66667	1.732050808	72168.78365
5	7	35714.28571	2.076521397	74161.47845
6	8	31250	2.414213562	75444.17382
7	9	27777.77778	2.747477419	76318.81721
8	10	25000	3.077683537	76942.08843
9	11	22727.27273	3.405687239	77401.9827
10	12	20833.33333	3.732050808	77751.05849
11	13	19230.76923	4.057159486	78022.2978
12	14	17857.14286	4.381286268	78237.25478
13	15	16666.66667	4.704630109	78410.50182
14	16	15625	5.027339492	78552.17956
15	17	14705.88235	5.349527506	78669.52214
16	18	1388.88889	5.67128182	78767.80305
17	19	13157.89474	5.992671459	78850.94024
18	20	12500	6.313751515	78921.89393
19	30	8333.333333	9.514364454	79286.37045
20	40	6250	12.70620474	79413.7796
21	50	5000	15.89454484	79472.72422
22	60	4166.666667	19.08113669	79504.7362
23	70	3571.428571	22.26673006	79524.03592
24	80	3125	25.45169958	79536.56119
25	90	2777.777778	28.63625328	79545.14801
26	100	2500	31.82051595	79551.28988
27	1000	250	318.308839	79577.20975
28	10000	25	3183.098757	79577.46893
29	100000	2.5	31830.98861	79577.47152
30	1000000	0.25	318309.8862	79577.47154
31	10000000	0.025	3183098.862	79577.47155
32	100000000	0.000025	3183099479	79577.48699

Now that I have a spreadsheet of the areas of the different polygons, I can construct a graph. The graph will involve the number of sides (n) and the total area of the polygon.



As the graph shows, the area increases quite rapidly at first, but then smoothes out as the number of sides (n) increases. This graph has an asymptote (a line that the curve approaches but never quite reaches at 80 000.

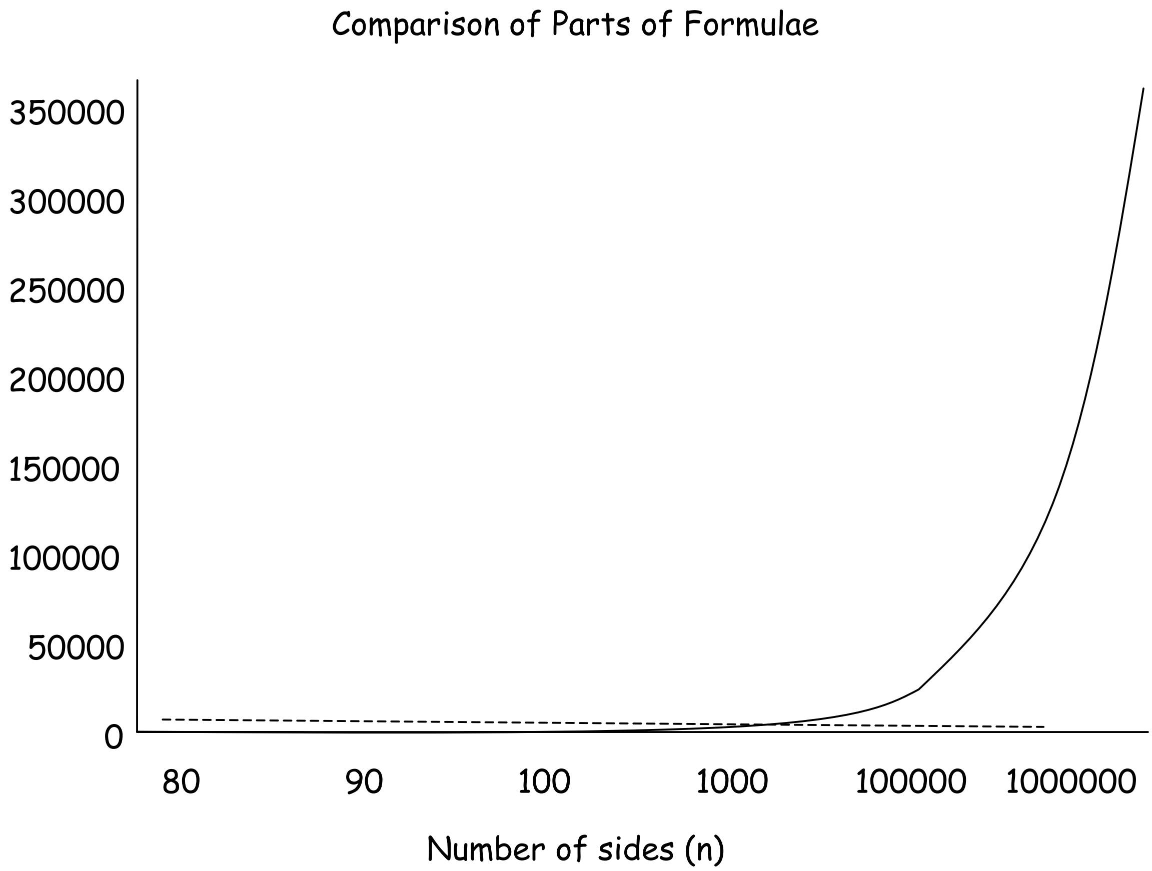
Therefore, as the number of sides (n) on the polygon increases, the area increases as well. Therefore, a polygon with infinite sides, which is a circle, will give the maximum area possible for a perimeter, or circumference in this case, of 1000m. So, a circle (infinite sided polygon) will give the maximum area possible.

Now that I have stated that a circle will give the maximum area possible, I can now try to explain why as the number of sides increase, the area increases as well. To demonstrate this, I will use my formula:

$$\text{Area} = \frac{500^2}{n} \times \tan\left(90 - \frac{180}{n}\right)$$

As n increases, therefore, the $(500^2)/n$ part of the formula will decrease because 500^2 is being divided by a bigger number, and therefore into smaller fractions. In the second part of the formula, tan $(90 - 180/n)$, as n increases, the total value of this part increases. This is because 180 is being divided into smaller fractions by the increasing n; as the 180/n decreases because of this, the value is becoming closer to 90. This is because I am subtracting the 180/n from the 90, and, when 180/n decreases, the total value (in the brackets) will therefore increase.

However, this still does not fully explain why the area increases overall. So far, I have established that one part of the formula is increasing while the other is decreasing. The reason why the area increases is because the tan (90-180/n) part of the formula is increasing at a faster rate than the $(500^2)/n$ is decreasing. This can be represented graphically:



As shown on the graph, the solid line (tan (90 - 180/n)) is a lot steeper than the dashed line (250000/n). Therefore the area will increase when the number of sides (n) increases.

So far, I know that an infinitely sided polygon (as circle) gives the largest area. The formula for calculating the area of a circle is:

$$\pi r^2$$

Theoretically, I should be able to relate the polygon formula and the circle. Since they are both polygons.

Area of polygon = 500^2 /n x tan (90-180/n)

Area of circle = πr^2

I need to change the circle formula into something else to help me link the two formulae.

Circumference=$2\pi r$
$1000=2\pi r$
$500/\pi=r$

Therefore:
Area = $\pi \times 500^2/\pi^2$
Area = $500^2/\pi$

Polygon=500^2/n x tan(90 - 180/n)
Circle = $500^2/\pi$

Polygon = 500^2 x 1/n x tan (90 - 180/n)
Circle = $500^2 \times 1/\pi$

I can cancel out the 500^2 so that:

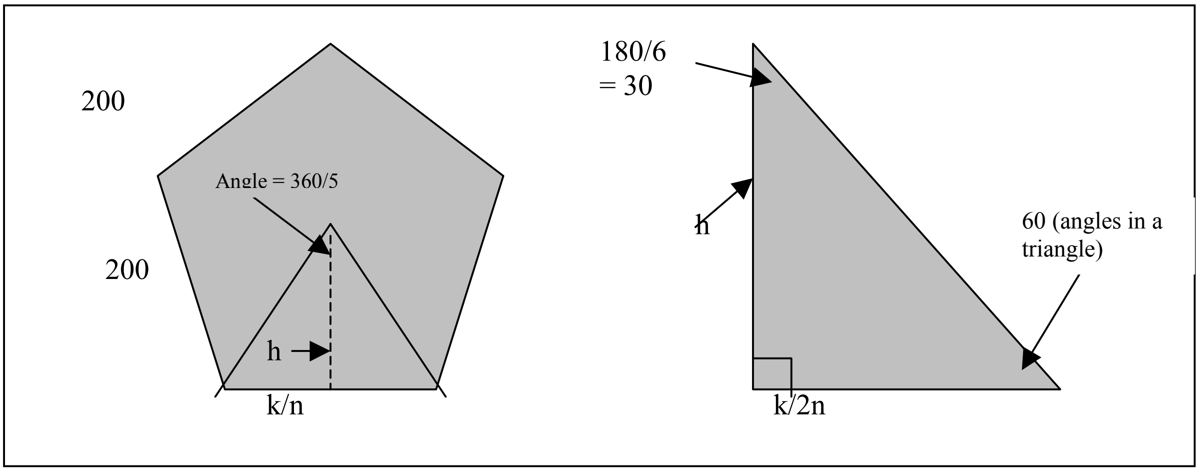
$1/\pi$ = 1/n x tan (90 - 180/n)
n = π tan(90 - 180/n)

n/tan(90- 180/n)= π

This formula is more accurate when n is increased to large numbers: 10 000, for example.

10 000/tan (90 - 180/10 000)= 3.141592757
compared with π: 3.141592654

The above formula gives a link between the polygon formula and the formula for a circle, using pi (π). However, my goal was to link directly the two formulas. I cannot do this because I have no values for the radius of the circle in my formula. It would be useful if I could create another formula, which could incorporate πr^2.



The formula I had previously, i.e. Area = $500^2/n \times \tan(90 - 180/n)$, used the angle adjacent to the base of the triangle. This angle requires quite a complex expression when it is applied to the formula. The angle at the top of the triangle, θ or theta, is the centre angle of the polygon or circle. This angle requires a very simple algebraic term:

Angle $\theta = 180/n$

This is a lot simpler than 90 - 180/n, and so this is the angle that I shall use to create an alternative formula to calculate the area of a polygon or circle.

The first part of constructing this formula will be the calculation of the area of the triangle, using trigonometry.

Tan θ = opposite/adjacent
Tan $\theta = (k/2n)/h$

Therefore,

$$h = \frac{k/2n}{\tan\theta}$$

$$h = \frac{k/2n}{\tan 180/n}$$

As before, I can convert 180/n into radians.

Therefore:

$$h = \frac{k/2n}{\tan \pi/n}$$

I now have the perpendicular height formula for the triangle. I can now construct a formula for the area of any polygon.

Area = Number of sides x area of triangle (1/2 base x perpendicular height)

$$\text{Area} = n \times \frac{k}{2n} \times \frac{1}{\tan \pi / n} \times \frac{k}{2n}$$

I see that I have two obvious like terms that I can simplify:

Area = $n \times (k/2n)^2 \times 1/\tan \pi/n$

I will test the formula once:

Area = $10 \times (1000/20)^2 \times 1/\tan \pi/10$
Area = 76942.08843

Now that I have this formula, I can try to link it directly to πr^2:

To help me do this, I researched the functions and theories about the tangent function. I researched in the following books:

Heinemann Pure Mathematics 1
Essential Mathematics for A-level
Mathematical Models

Whilst studying these books, I found a theory suggesting that when n approaches infinity, tan θ approaches θ itself. This can be written:

$N \cong \infty$
$\text{Tan } \theta \cong \theta$

In light of this, I can now alter my formula so that tan will disappear. However, this formula will only be true when n is approaching infinity (∞).

$$\text{Area} = n \times (k/2n)^2 \times \frac{1}{\pi / n}$$

$= n \times (k/2n)^2 \times n/\pi$

$$= n \times \frac{k^2}{4n^2} \times n/\pi$$

$= \frac{k^2}{4n} \times n/\pi$

$= \frac{k^2}{4} \times 1/\pi$

I can now substitute k^2 (which is the circumference squared) for $(2\ \pi r^2)$ which is $4\pi^2 r^2$:

$= \frac{4 \times \pi^2 \times r^2}{4} \times 1/\pi$

$= \pi^2 \times r^2 \times 1/\pi$

$= \pi r^2$

Conclusion

I have now shown a direct link between the formula for the area of the polygon and the formula for the area of the circle. The second formula that I used for calculating the area of polygons was the superior one, because I could relate it to πr^2. It involved very few numerical values, as most of the values were in algebraic terms, and I think this was the key to being able to relate the formula to πr^2.

I finally conclude that the circle (infinitely sided polygon) gives the maximum area possible with a perimeter, or circumference in this case, of 1000 metres.

There will probably be other formulae that also calculate the area of a polygon, but I found that the formulae I used were the easiest to work out and also to do other calculus work on them.

As a result of my investigation, I have achieved the following:
Investigated different polygons and their areas and calculated a formula based on ideas that resulted from this.
Drew graphs demonstrating certain ideas and spreadsheets which calculated areas using formulae.

Based on many factors, I stated that a circle produced the largest area possible with a perimeter (or circumference) of 1000 metres.
Researched new ideas and theories; I then used these ideas to calculate another formula which could be related to πr^2.

Strand 1 Mark 8

The candidate has moved from a trigonometrical approach to developing an overall general formula for an '*n*'-sided polygon. This has been used to generate (with a spreadsheet) the correct results. The candidate then uses 'radian' measure to obtain the general formula in another form and uses both to consider the 'limiting' case.

Strand 2 Mark 8

The candidate has presented a concise argument as to why the circle gives the maximum area. This has been approached, with good supporting algebra, by considering the formula for an *n*-sided polygon as '*n*' increases in the limiting case.

Strand 3 Mark 8

The candidate has offered a rigorous argument to support the work throughout.

The work in this section is taken from the task 'Layers'.

So since the formula works and this sequence is the triangle numbers the number of arrangements for 8, 9 and 10 squares are 28, 36 and 45 respectively.

So the formula for one layer containing two less cubes than the size of the grid is $U_n = \frac{1}{2}n(n - 1)$.

From here on the layers added contain one less cube than the previous one. So the second layer has one less cube than the first layer. This was the case in question 4 and the number of arrangements of a layer having one cube less than the previous one is actually the number of cubes in the previous layer. So if there are n squares on the grid and n – 2 cubes on the first layer the number of arrangements of the cubes in the second layer is n – 2 for each of the arrangements of the first layer. So the total number of arrangements for two layers is $\frac{1}{2}n(n - 1)(n - 2)$.

Proof using notation: If the above formula is correct the arrangements for these grid sizes should be:

$U_4 = 2 \times 3 \times 2 = 12$

$U_5 = 2.5 \times 4 \times 3 = 30$

$U_6 = 3 \times 5 \times 4 = 60$

For 4 squares

$A_0B_0C_1$, $A_0B_0D_1$, $A_0C_0B_1$, $A_0C_0D_1$, $A_0D_0B_1$, $A_0D_0C_1$

$B_0C_0A_1$, $B_0C_0D_1$, $B_0D_0A_1$, $B_0D_0C_1$

$C_0D_0A_1$, $C_0D_0B_1$

12 arrangements

For 5 squares

A_0: B_0C_1, B_0D_1, B_0E_1, C_0B_1, C_0D_1, C_0E_1, D_0B_1, D_0C_1, D_0E_1, E_0B_1, E_0C_1, E_0D_1

B_0: C_0A_1, C_0D_1, C_0E_1, D_0A_1, D_0C_1, D_0E_1, E_0A_1, E_0C_1, E_0D_1

C_0: D_0A_1, D_0B_1, D_0E_1, E_0A_1, E_0B_1, E_0D_1

D_0: E_0A_1, E_0B_1, E_0D_1

30 arrangements

For 6 squares

A_0: B_0C_1, B_0D_1, B_0E_1, B_0F_1, C_0B_1, C_0D_1, C_0E_1, C_0F_1, D_0B_1, D_0C_1, D_0E_1, D_0F_1, E_0B_1, E_0C_1, E_0D_1, E_0F_1, F_0B_1, F_0C_1, F_0D_1, F_0E_1

B_0: C_0A_1, C_0D_1, C_0E_1, C_0F_1, D_0A_1, D_0C_1, D_0E_1, D_0F_1, E_0A_1, E_0C_1, E_0D_1, E_0F_1, F_0A_1, F_0C_1, F_0D_1, F_0E_1

C_0: D_0A_1, D_0B_1, D_0E_1, D_0F_1, E_0A_1, E_0B_1, E_0D_1, E_0F_1, F_0A_1, F_0B_1, F_0D_1, FoE_1

D_0: E_0A_1, E_0B_1, E_0C_1, E_0F_1, F_0A_1, F_0B_1, F_0C_1, F_0E_1
E_0: F_0A_1, F_0B_1, FoC_1, F_0D_1
60 arrangements

The predicted values of arrangements were correct so the formula for the number of arrangements when there are two less cubes in the first layer than the number of squares and one less cube in the second layer than in the first one is
$U_n = \frac{1}{2}(n - 1)(n - 2)$ where n is the size of the grid.

Using the same logic as above when a third layer is added the number of arrangements will be described by the formula $\frac{1}{2}n(n - 1)(n - 2)(n - 3)$. $n - 3$ is the number of arrangements for the third layer on top of the second. Using this the number of arrangements of cubes on a 2 x 3(6) grid with three layers will be
$3 \times 5 \times 4 \times 3 = 180$.

NB: Instead of writing all the arrangements I will find only how many there are if only the third layer cube moves. The first two cubes I already know the number of arrangements because I found it above. For two layers there are 60 arrangements.

For 6 squares-3 layers (two layer constant is $A_0B_0C_1$)
$A_0B_0C_1D_2$, $A_0B_0C_1E_2$, $A_0B_0C_1F_2$
Multiplying this by 60 gives an answer of 180 so the formula works.

So the formula for 3 layers if the first has two less cubes than the squares and the other two one less cube than the previous one the formula is
$\frac{1}{2}n(n - 1)(n - 2)(n - 3)$.

Proving the formula for the first three layers I can now find the following:

For 4 layers the formula is $\frac{1}{2}n(n - 1)(n - 2)(n - 3)(n - 4)$.
For *5* layers the formula is $\frac{1}{2}n(n - 1)(n - 2)(n - 3)(n - 4)(n - 5)$.
For 6 layers the formula is $\frac{1}{2}n(n - 1)(n - 2)(n - 3)(n - 4)(n - 5)(n - 6)$.

The general formula for this can be derived by combining $\frac{1}{2}n(n - 1)$ for the first layer with the formula $n!/(n - x)!$. This time however the number of arrangements for the layers above the first one is actually $(n - 2)!$ since there are 2 cubes less in the first layer. For the same layers the layers not present is $n - (x + 1)$ because we have to subtract the first layer. So the formula for the layers above the first one is $(n - 2)!/[n - (x + 1)]!$.

And the formula for all layers is $\frac{1}{2}n(n-1)(n-2)!/[n-(x+1)]!$. $(n-1)(n-2)$ can be simplified into $(n-1)!$ since this also includes $(n-2)!$ and also into $n!$ So the final formula is $U_n = n!/2[n-(x+1)]!$ where n is the size of the grid and x is the number of layers.

<u>Proof using previous results:</u>
For 1 layer on a 2 x 3 grid $U_n = (6 \times 5 \times 4 \times 3 \times 2 \times 1)/(2 \times 4 \times 3 \times 2 \times 1) = 15$ (correct)
For 2 layers on a 5 size grid $U_n = (5 \times 4 \times 3 \times 2 \times 1)/(2 \times 2 \times 1) = 30$ (correct)
For 3 layers on a 2 x 3 grid $U_n = (6 \times 5 \times 4 \times 3 \times 2 \times 1)/(2 \times 2 \times 1) = 180$ (correct)

<u>Conclusion:</u> The relationship between the size of the grid (n) and the number of layers (x) where the first layer has two less cubes than the number of squares and the rest layers have one less cube each time is described by the formula $U_n = n!/2[n-(x+1)]!$

When there are two cubes less in every layer than the previous one.
So far I found that if the first layer has two less cubes than the number of squares the formula is $\frac{1}{2}n(n-1)$. So I predict that if another layer with two less cubes than the first is added the formula for the second layer will be $\frac{1}{2}(n-2)(n-3)$. So for both layers the formula is $\frac{1}{2}n(n-1)\frac{1}{2}n(n-2)(n-3)$ or $\frac{1}{4}n(n-1)(n-2)(n-3)$.

E.g. if we had a 2 x 3 grid and placed 2 layers on it the first layer would have two less cubes and 15 different arrangements as I found before. For the second layer two cubes sit on 4 cubes of the first layer. If we consider this to be 2 cubes on a 2 x 2 grid these are the different arrangements:
A_0B_0, A_0C_0, A_0D_0
B_0C_0, B_0D_0
C_0D_0
Using the formula-$U_6 = \frac{1}{2} \times 4 \times 3 = 6$
So there are 6 arrangements for the second layer and if we multiply this by the 15 arrangements of the first layer we get a total of 90 arrangements.
Using the formula $U_6 = 6 \times 5 \times 4 \times 3/4 = 90$
So we can see that the formula works for this case.

If we had a 5 square grid the number of arrangements for the first layer is 10 as I found before and the second layer is like 1 cube on a 3 size grid. The arrangements are 3 for the second layer so the total is 3 x 10 = 30
Using the formula $U_5 = 5 \times 4 \times 3 \times 2/4 = 30$ (correct)

If a third layer is added, using the same logic the number of arrangements of this layer can be found by the formula $\frac{1}{2}(n - 4)(n - 5)$.
So the formula to find all arrangements is
$\frac{1}{2}n(n - 1)\frac{1}{2}(n - 2)(n - 3)\frac{1}{2}(n - 4)(n - 5)$ or $\frac{1}{8}n(n - 1)(n - 2)(n - 3)(n - 4)(n - 5)$.

On a 4 x 2 grid (using formulas found before)
1st layer – $\frac{1}{2}(8 \times 7) = 28$ arrangements
2nd layer – $\frac{1}{2}(6 \times 5) = 15$ arrangements
So for the first two layers there are 28 x 15 = 420 arrangements.
The third layer will be like 2 cubes lying on a 2 x 2 grid so the arrangements are 6 as I found before. The total therefore is
420 x 6 = 2520 arrangements.
Using formula: $U_8 = 8 \times 7 \times 6 \times 5 \times 4 \times 3/8 = 2520$
We can see therefore that the formula works.

So using the same logic:
For 4 layers $U_n = \frac{1}{16}n(n - 1)(n - 2)(n - 3)(n - 4)(n - 5)(n - 6)(n - 7)$
For 5 layers $U_n =$
$$\frac{1}{32}n(n - 1)(n - 2)(n - 3)(n - 4)(n - 5)(n - 6)(n - 7)(n - 8)(n - 9)$$
For 6 layers $U_n =$
$$\frac{1}{64}n(n - 1)(n - 2)(n - 3)(n - 4)(n - 5)(n - 6)(n - 7)(n - 8)(n - 9)(n - 10)(n - 11)$$

General formula

If we take n to be 14

For 4 layers $U_{14} = \frac{1}{16}(14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7)$ or $14!/6! \times 16$
For 5 layers $U_{14} = \frac{1}{32}(14 \times 13 \times 12 \times 111 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5)$
or $14!/4! \times 32$
For 6 layers $U_{14} = \frac{1}{64}(14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3)$
or $14!/2! \times 64$
The shaded parts 6!, 4! and 2! above can be obtained in terms of n and x by $(n - 2x)!$
Also the last terms (16, 32, 64) are found using 2 to the power of x.
The general formula therefore is $\mathbf{U_n = n!/(n - 2x)!2^x}$

Proof using previous results

U_6 with 2 layers = $6 \times 5 \times 4 \times 3 \times 2 \times 1/2 \times 1 \times 4 = 90$ arrangements (correct)

U_8 with 3 layers = 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1/2 x 1 x 8 = 2520 arrangements (correct)

Conclusion: The relationship between he size of the grid (n) and the number of layers (x) where each layer has two cubes less is described by the formula $U_n = n!/(n - 2x)!2^x$

When there are 3 less cubes in the first layer

For the first layer

When there are 4 squares in the grid and the first layer has three less cubes i.e. one then the number of arrangements is 4 since one cube can be in four different places.

On a 5 size grid

$A_0B_0C_0$, $A_0B_0D_0$, $A_0B_0E_0$
$A_0C_0D_0$, $A_0C_0E_0$
$A_0D_0E_0$
$B_0C_0D_0$, $B_0C_0E_0$
$B_0D_0E_0$
$C_0D_0E_0$
10 arrangements

On a 6 size grid

$A_0B_0C_0$, $A_0B_0D_0$, $A_0B_0E_0$, $A_0B_0F_0$
$A_0C_0D_0$, $A_0C_0E_0$, $A_0C_0F_0$
$A_0D_0E_0$, $A_0D_0F_0$
$A_0E_0F_0$
$B_0C_0D_0$, $B_0C_0E_0$, $B_0C_0F_0$
$B_0D_0E_0$, $B_0D_0F_0$
$B_0E_0F_0$
$C_0D_0E_0$, $C_0D_0F_0$
$C_0E_0F_0$
$D_0E_0F_0$
20 arrangements

On a 7 size grid

$A_0B_0C_0$, $A_0B_0D_0$, $A_0B_0E_0$, $A_0B_0F_0$, $A_0B_0G_0$
$A_0C_0D_0$, $A_0C_0E_0$, $A_0C_0F_0$, $A_0C_0G_0$
$A_0D_0E_0$, $A_0D_0F_0$, $A_0D_0G_0$
$A_0E_0F_0$, $A_0E_0G_0$
$A_0F_0G_0$

$B_0C_0D_0$, $B_0C_0E_0$, $B_0C_0F_0$, $B_0C_0G_0$
$B_0D_0E_0$, $B_0D_0F_0$, $B_0D_0G_0$
$B_0E_0F_0$, $B_0E_0G_0$
$B_0F_0G_0$
$C_0D_0E_0$, $C_0D_0F_0$, $C_0D_0G_0$
$C_0E_0F_0$, $C_0E_0G_0$
$C_0F_0G_0$
$D_0E_0F_0$, $D_0E_0G_0$
$D_0F_0G_0$
$E_0F_0G_0$
35 arrangements

At this point I realised that the differences between the number of arrangements are the triangle numbers (3 6 10 15) the formula of which is $\frac{1}{2}n(n - 1)$ so I believe that the formula for this problem will contain the factor $\frac{1}{2}n(n - 1)$.

n	4	5	6	7
$\frac{1}{2}n(n - 1)$	6	10	15	21
second factor	4/6	1	4/3	5/3
U_n	4	10	20	35

This is how the number of arrangements is derived if we multiply n by $\frac{1}{2}n(n - 1)$. Looking at the second factor I can write it as 4/6 6/6 8/6 10/6 or in general (2n – 4)/6. This can be simplified into (n – 2)/3. So I can now predict that the formula for the first layer when it has 3 less cubes than the size of the grid is $\frac{1}{2}n(n - 1)^{(n - 2)}/3$ or $\frac{1}{6}n(n - 1)(n - 2)$.

Proof using previous results

For a 4 size grid there are 4 x 3 x 2/6 = 4 arrangements (correct)
For a *5* size grid there are 5 x 4 x 3/6 = 10 arrangements (correct)
For a 7 size grid there are 7 x 6 x 5/6 = 35 arrangements (correct)

Conclusion: The relationship between the size of the grid (n) and the number of arrangements of the first layer when it has three less cubes than the number of squares is described by the formula $\frac{1}{6}n(n - 1)(n - 2)$.

If now we add a second layer having one less cube than the first the formula for it should be n – 3.
So the formula for both layers is $\frac{1}{6}n(n - 1)(n - 2)(n - 3)$.

On a 2 x 3 grid the number of arrangements for the first layer is 20. The second layer is two cubes on top of three so there are 3 arrangements for each of the 20, a total of 60. Using the formula: 6 x 5 x 4 x 3/6 = 60.

So if a third layer is added with one cube less than the second the formula should be $\frac{1}{6}n(n - 1)(n - 2)(n - 3)(n - 4)$. I therefore notice that the general formula is $n!/6[n - (x + 2)]!$

<u>Proof:</u> If we had a *5* size grid with 2 layers the first of which has 2 cubes and the other 1 the number of arrangements should be 5!/6 x 1! = 20. We can also find this if we said that the first layer has 10 arrangements (found before) and the second has 2 (1 cube on top of two). So the formula works for this case.
If we had a 2 x 4 grid and 4 layers the number of arrangements for the first layer would be 8 x 7 x 6/6 = 56. Then for the other three it is as if we had three layers on a 5 size grid each having one cube less every time. The formula for this was found in question 4 and the number of arrangements is 5!/(5 – 3)! = 60. So the total number of arrangements is 56 x 60 = 3360.
Using the general formula 8!/6(8 – 6)! = 3360.

<u>Conclusion:</u> **The relationship between the number of arrangements and the size of the grid n, when we have x layers the first of which with n-3 cubes and t rest one cube less every time is described by the formula $\frac{1}{6}n(n - 1)(n - 2)$.**

When there are 3 less cubes in every layer.

Just like in the case where we had two less cubes every time I predict that the formula for this case will be a combination of the formula for the first layer which is $\frac{1}{6}n(n - 1)(n - 2)$. When there are two layers with three less cubes every time the formula will be $\frac{1}{6\times6}n(n - 1)(n - 2)(n - 3)(n - 4)(n - 5)$.

<u>Proof:</u> If we had a 4 x 2 grid the arrangements for the first layer would be 8 x 7 x 6/6 = 56. For the second layer we have 2 cubes on top of five so 5 x 4 x 3/6 = 10 arrangements.
So the total number of arrangements is 56 x 10 = 560.
Using the formula the number of arrangements is 8 x 7 x 6 x 5 x 4 x 3/36 = 560. So the formula is correct.

In the same way if a third layer with three less cubes is added the formula would be $\frac{1}{6\times6\times6}n(n - 1)(n - 2)(n - 3)(n - 4)(n - 5)(n - 6)(n - 7)(n - 8)$

So the general formula is $n!/(n - 3x)!6^x$

<u>Proof using previous results</u>: Previously I found that for two layers on a 2 x 4 grid there are 560 arrangements. Using the formula $8!/2 \times 1 \times 6^2 = 560$ (correct).
The formula should also work for one layer so one layer on a 7 size grid there are 7!/4! x 6 = 35 arrangements (correct).

Conclusion: The relationship between the number of arrangements and the size of the grid n, when we have x layers each having 3 less cubes every time is described by the formula $n!/(n - 3x)!6^x$.

When there are four less cubes in the first layer.

Theoretically there is one arrangement on a 2 x 2 grid. When there is one layer on a 5 size grid we have 5 arrangements since there is only one cube.

On a 2 x 3(6) grid (showing the two cubes meaning the rest are empty)
$\mathbf{A_1}$: B_1, C_1, D_1, E_1, F_1
$\mathbf{B_1}$: C_1, D_1, E_1, F_1
$\mathbf{C_1}$: D_1, E_1, F_1
$\mathbf{D_1}$: E_1, F_1
$\mathbf{E_1}$: F_1
15 arrangements

On a 7 size grid
$\mathbf{A_1B_1}$: C_1, D_1, E_1, F_1, G_1
$\mathbf{A_1C_1}$: D_1, E_1, F_1, G_1
$\mathbf{A_1D_1}$: E_1, F_1, G_1
$\mathbf{A_1E_1}$: F_1, G_1
$\mathbf{A_1F_1}$: G_1
$\mathbf{B_1C_1}$: D_1, E_1, F_1, G_1
$\mathbf{B_1D_1}$: E_1, F_1, G_1
$\mathbf{B_1E_1}$: F_1, G_1
$\mathbf{B_1F_1}$: G_1
$\mathbf{C_1D_1}$: E_1, F_1, G_1
$\mathbf{C_1E_1}$: F_1, G_1
$\mathbf{C_1F_1}$: G_1
$\mathbf{D_1E_1}$: F_1, G_1
$\mathbf{D_1F_1}$: G_1
$\mathbf{E_1F_1}$: G_1
35 arrangements

On a 2 x 4(8) grid

$A_1B_1C_1$: D_1, E_1, F_1, G_1, H_1
$A_1B_1D_1$: E_1, F_1, G_1, H_1
$A_1B_1E_1$: F_1, G_1, H_1
$A_1B_1F_1$: G_1, H_1
$A_1B_1G_1$: H_1
$A_1C_1D_1$: E_1, F_1, G_1, H_1
$A_1C_1E_1$: F_1, G_1, H_1
$A_1C_1F_1$: G_1, H_1
$A_1C_1G_1$: H_1
$A_1D_1E_1$: F_1, G~, H_1
$A_1D_1F_1$: G_1, H_1
$A_1D_1G_1$: H_1
$A_1E_1F_1$: G_1, H_1
$A_1E_1G_1$: H_1
$A_1F_1G_1$: H_1
$B_1C_1D_1$: E_1,F_1, G_1,H_1
$B_1C_1E_1$: F_1, G_1, H_1
$B_1C_1F_1$: G_1, H_1
$B_1C_1G_1$: H_1
$B_1D_1E_1$: F_1, G_1, Lit
$B_1D_1F_1$: G_1, H_1
$B_1D_1G_1$: H_1
$B_1E_1F_1$: G~, H_1
$B_1E_1G_1$: H_1
$B_1F_1G_1$: H_1
$C_1D_1E_1$: F_1, G_1, H_1
$C_1D_1F_1$: G_1, H_1
$C_1D_1G_1$: H_1
$C_1E_1F_1$: G_1, H_1
$C_1E_1G_1$: H_1
$C_1F_1G_1$: H_1
$D_1E_1F_1$: G_1, H_1
$D_1E_1G_1$: H_1
$D_1F_1G_1$: H_1
$E_1F_1G_1$: H_1
70 arrangements

Up to now we have the sequence 1 5 15 35 70. Again, as in the case where we had 3 less layers I notice that the differences in this sequence are the arrangements of the previous parts when we had 3 less layers. So the difference table is

1		5		15		35		70		126		210
	4		10		20		35		56		84	
		6		10		15		21		28		
			4		5		6		7			

Since the differences between the terms are the number of arrangements for three layers I predict that one of the factors in the formula will be $\frac{1}{6}n(n-1)(n-2)$.

n	4	5	6	7
$\frac{1}{6}n(n-1)(n-2)$	4	10	20	35
second factor	1/4	1/2	3/4	1
U_n	1	5	15	35

This is how the number of arrangements is derived. The second factor can be written as 1/4 2/4 3/4 4/4 or (n – 3)/4. So the final formula is $\frac{1}{6\times4}n(n-1)(n-2)(n-3)$.

<u>Proof:</u> On a 2 x 4 grid U_8 = 8 x 7 x 6 x 5/24 = 70 (correct).
So the formula for the first layer when there are 4 less cubes than the number of squares is $\frac{1}{24}n(n-1)(n-2)(n-3)$.

If a second layer is added having one less cube than the first the formula for it will be n – 4 so the formula for all of the arrangements is $\frac{1}{24}n(n-1)(n-2)(n-3)(n-4)$. Similarly if a third one is added the formula will be $\frac{1}{24}n(n-1)(n-2)(n-3)(n-4)(n-5)$. So the general formula if x number of layers are added with the first having 4 less cubes is n!/24[n – (x + 3)]!

<u>Proof:</u> On a 2 x 4 grid with 2 layers the first has 70 arrangements and the second 4. This gives a total of 70 x 4 = 280 arrangements.
Using the formula 8!/24 x 3! = 280.

Conclusion: The relationship between the number of arrangements and the size of the grid n, when we have x layers with the first having 4 less cubes than the grid size and the rest 1 less every time is n!/24[n – (x + 3)]!

When there are 4 less cubes in every layer.

Just like in the case where we had three less cubes every time I predict that the formula for this case will be a combination of the formula for the

first layer which is $\frac{1}{24}n(n-1)(n-2)(n-3)$. When there are two layers with four less cubes every time the formula will be
$\frac{1}{24 \times 24}n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)(n-7)$.

<u>Proof:</u> If we had 2 layers on a 2 x 5(10) grid the number of arrangements for the first layer would be 10 x 9 x 8 x 7/24 = 210. Then for the second layer we have 2 cubes on top of 6 so as we found before there are 15 arrangements. So the total number of arrangements is 3150.
Using the formula U_{10} = 10 x 9 x 8 x 7 x 6 x 5 x 4 x 3/24 x 24 = 3150. So the formula is correct. Similarly if a third layer is added the formula to find the number of arrangements is
$\frac{1}{24 \times 24 \times 24}n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)(n-7)(n-8)(n-9)(n-10)(n-11)$.
So the general formula is $n!/(n-4x)!24^x$.

<u>Proof using previous results</u>: I found previously that 2 layers on a 10 size grid can have 3150 arrangements.
Using the formula $U_{10} = 10!/2! \times 24^2 = 3150$ (correct)

Conclusion: The relationship between the number of arrangements and the size of the grid n, when we have x layers each having 4 less cubes than the previous one is $n!/(n-4x)!24^x$.

Summary: this is what we have up to now.

With the first layer having y cubes less and the rest 1

y	Formula
1	n!/(n – x)!
2	n!/2[n – (x + 1)]!
3	n!/6[n – (x + 2)]!
4	n!/24{n – (x + 3)]!

Prediction for formula when y is 5: As I noticed above the factors 2 6 24 by which we divide n! are derived if we multiply the previous one (y – 1) by y. So this factor when y is 5 will be equal to 24 x 5= 120. I also notice that each time we divide by one less (x, x + 1, x + 2 etc) so the formula when y is 5 should be n!/120[n – (x + 4)]!

<u>Proof:</u> Previously I noticed that the differences of the sequence of arrangements of the first layer are the number of arrangements of the first layer when we take one less cube out. So when we take 5 cubes away from the first layer and there are 6 arrangements on a 2 x 3 grid the rest

are found using this difference table.

1		6		21		56		126		252		462
	5		15		35		70		126		210	

Using the formula: $U_8 = 8!/120 \times 3! = 56$ (correct)
$U_{10} = 10!/120 \times 5! = 252$ (correct)

Similarly the formula when we take 6 cubes away from the first layer is $n!/720[n - (x + 5)]!$

General formula: the general formula for the number of arrangements U when x layers are laid on an n size grid with the first layer having y cubes less than the number of squares on the grid is $n!/y![n - (x + y - 1)]!$

Proof using previous result:
On a 2 x 4 grid with 2 layers the first of which has 4 less cubes and the second I we have 280 arrangements.
Using the formula $8!/4! \times 3! = 280$ (correct).

Summary: this is what we have up to now.

With all the layers having y cubes less and the previous layer.

y	**Formula**
1	$n!/(n - x)!$
2	$n!/(n - 2x)!2^x$
3	$n!/(n - 3x)!6^x$
4	$n!/(n - 4x)!24^x$

Again the second factor by which we divide n! is y! so the general formula should be $n!/(n - yx)!\, y!^{x}$

Proof using previous results: on an 11 size grid with one layer having 5 cubes less there are 462 arrangements. Using the formula $U_{11} = 11!/(11 - 5)! \times 5! = 462$. On a 2 x 5(10) grid with 2 layers each having 4 cubes less the number of arrangements is 3150. Using the formula $U_{10} = 10!/(10 - 8)! \times 4!^2 = 3150$ (correct).

Conclusion: The relationship between the size of the grid n and the number of layers x:
1) When the first layer has y less cubes and the rest 1 than the previous one is described by the formula $n!/y![n - (x + y - 1)]!$
2) When all layers have y less cubes by the formula $n!/(n - yx)!\, y!^{x}$

Strand 1 Mark 8

The candidate has obtained the solutions to many cases where the number of cubes on each layer, the number of layers and the number of cubes less each time can vary. This has been done using factorial notation supported by good arguments.

Strand 2 Mark 8

The candidate has offered an elegant argument and has obtained the correct overall generalisation supported by good mathematics.

Strand 3 Mark 8

The candidate has justified the work throughout. The justification is sound and fully supports the work at this level.

Procedures for moderation of internal assessment

All centres will receive Optically-read Teacher Examiner Mark Sheets (OPTEMS) for each coursework component.

Centres will have the option of:

EITHER

recording marks on an Optically-read Teacher Examiner Mark Sheet (OPTEMS), Section 1

OR

recording marks on computer for transfer to Edexcel by means of Electronic Data Interchange (EDI), Section 2.

Sections 3 and 4 apply whichever option is selected and deal with Coursework Record Sheets and the sample of work required for moderation.

1 Centres using OPTEMS

1.1 OPTEMS will be pre-printed on three-part stationery with unit and paper number, centre details and candidate names in candidate number order. A number of blank OPTEMS for candidates not listed will also be supplied.

The top copy is designed so that the marks can be read directly by an Optical Mark Reader. It is important therefore to complete the OPTEMS carefully in accordance with the instructions below. **Please do not fold or crease the sheets**.

1.2 Before completing the OPTEMS please check the subject, paper and centre details, to ensure the correct sheet is being completed.

1.3 All candidates entered by the deadline date will be listed on the OPTEMS, except those carrying forward their centre-assessed marks from the previous year. Such candidates will be listed on a separate OPTEMS coded T for Transferred. Any OPTEMS coded T should be checked, signed to confirm the transfer, and the top copy returned to Edexcel. No mark should be entered.

NB: Any candidate who wishes to transfer a moderated coursework mark must be entered for option T and a form CF99 completed.

1.4 Late entries will need to be added in pencil either in additional spaces on the pre-printed OPTEMS or on one of the blank OPTEMS which will be supplied. Please note that full details of the centre, specification/unit, paper, candidates' names and candidate numbers must be added to ALL blank OPTEMS.

Candidates who have transferred between the tiers since the OPTEMS were printed should be coded as W on the incorrect OPTEMS and added to the bottom of the correct OPTEMS.

1.5 The OPTEMS should be completed **using an HB pencil.** Please ensure that you work on a firm flat surface and that figures written in the marks box go through to the second and third copies.

1.6 For each candidate, first ensure you have checked the arithmetic on the Coursework Record Sheet, then transfer the **Total Mark** to the box of the OPTEMS labelled 'Marks' for the correct candidate (Please see exemplar).

1.7 Encode the component mark on the right-hand side by drawing a line to join the two dots inside the ellipses on the appropriate marks. Clear, dark **HB pencil** lines must be made but they must not extend outside the ellipses on either side of the two dots. Take care to remember the trailing zeros for candidates scoring 10, 20 etc and the leading zero for single figures, as shown.

1.8 If you make a mistake rub out the incorrect marks completely. Amend the number in the marks box and in the encoded section, but **please remember to amend separately the second and third copies** to ensure that the correct mark is clear.

1.9 Every candidate listed on the OPTEMS must have either a mark or one of the following codes in the marks box.

a 0 (zero marks) should be entered only if work submitted has been found to be worthless. It should **not** be used where candidates have failed to submit work.

b ABS in the marks box and an A in the encoded section for any candidate who has been absent or has failed to submit any work, even if an aegrotat award has been requested.

c W should be entered in the marks box and the encoded section where the candidate has been withdrawn.

EXEMPLAR

Encoded section

Candidate Name	**Number**	**Marks**												
NEW ALAN* SP	3200	0	~~(•0•)~~ ~~(•0•)~~	(•10•) (•1•)	(•20•) (•2•)	(•30•) (•3•)	(•40•) (•4•)	(•50•) (•5•)	(•60•) (•6•)	(•70•) (•7•)	(•80•) (•8•)	(•90•) (•9•)	(•100•) (•A•)	(•200•) (•W•)
OTHER AMY* SP	3201	5	~~(•0•)~~ (•0•)	(•10•) (•1•)	(•20•) (•2•)	(•30•) (•3•)	(•40•) (•4•)	(•50•) ~~(•5•)~~	(•60•) (•6•)	(•70•) (•7•)	(•80•) (•8•)	(•90•) (•9•)	(•100•) (•A•)	(•200•) (•W•)
SMITH JOHN AW	3202	47	(•0•) (•0•)	(•10•) (•1•)	(•20•) (•2•)	(•30•) (•3•)	~~(•40•)~~ (•4•)	(•50•) (•5•)	(•60•) (•6•)	(•70•) ~~(•7•)~~	(•80•) (•8•)	(•90•) (•9•)	(•100•) (•A•)	(•200•) (•W•)
WATTS MARK* SP	3203	ABS	(•0•) (•0•)	(•10•) (•1•)	(•20•) (•2•)	(•30•) (•3•)	(•40•) (•4•)	(•50•) (•5•)	(•60•) (•6•)	(•70•) (•7•)	(•80•) (•8•)	(•90•) (•9•)	(•100•) ~~(•A•)~~	(•200•) (•W•)
JONES ANN* AW	3205	30	(•0•) ~~(•0•)~~	(•10•) (•1•)	(•20•) (•2•)	~~(•30•)~~ (•3•)	(•40•) (•4•)	(•50•) (•5•)	(•60•) (•6•)	(•70•) (•7•)	(•80•) (•8•)	(•90•) (•9•)	(•100•) (•A•)	(•200•) (•W•)
PATEL RAJ* AW	3206	27	(•0•) (•0•)	(•10•) (•1•)	~~(•20•)~~ (•2•)	(•30•) (•3•)	(•40•) (•4•)	(•50•) (•5•)	(•60•) (•6•)	(•70•) ~~(•7•)~~	(•80•) (•8•)	(•90•) (•9•)	(•100•) (•A•)	(•200•) (•W•)
WEST SARA SP	3207	W	(•0•) (•0•)	(•10•) (•1•)	(•20•) (•2•)	(•30•) (•3•)	(•40•) (•4•)	(•50•) (•5•)	(•60•) (•6•)	(•70•) (•7•)	(•80•) (•8•)	(•90•) (•9•)	(•100•) (•A•)	(•200•) ~~(•W•)~~

1.10 Where more than one teacher has assessed the work, the teachers' initials should be given to the right of each candidate's name as illustrated.

1.11 The authentication and internal standardisation statement on the OPTEMS must be signed. **Centres are reminded that it is their responsibility to ensure that internal standardisation of the marking has been carried out**.

Once completed and signed the three-part sets should then be divided and despatched, or retained as follows:

a **top copy** to be returned direct to Edexcel in the envelope provided **to be received by 1 May for the May/June examination series, and a date to be announced on the examination timetable for the winter examination series**. Please remember this form **must not be folded or creased**.

b **Second copy** to be sent **with the sampled coursework** as appropriate (see Section 4) to the moderator. The name and address of the moderator will either be printed on the OPTEMS or supplied separately.

c **Third copy** to be retained by the centre.

2 Centres using EDI

2.1 Marks must be recorded on computer and transmitted to Edexcel by **1 May for the May/June examination series, and a date to be announced on the examination timetable for the winter examination series**. They must be recorded in accordance with the specifications in the booklet 'Formats for the Exchange of Examination Related Data using Microcomputers'. Each mark has a status as well as a value. Status codes are:

V – valid non-zero mark recorded; candidate not pre-selected as part of the sample for moderation

S – valid non-zero mark recorded and candidate included in sample for moderation (refer to OPTEMS and Section 4)

Z – zero mark recorded for work submitted

N – no work submitted but candidate **not** absent

A – absent for component

M – missing mark; no information available about the candidate's previous performance

F– mark carried forward from a previous examination series. (If the mark status is 'F', then no mark follows.)

The OPTEMS provided will indicate, with asterisks, the candidates whose work is to be sampled, where this is pre-selected (see Section 4).

2.2 Printout

Centres are required to produce a printout of the centre-assessed marks and annotate it as described below, before forwarding it **together with the sampled coursework** as appropriate (see Section 4) to the moderator, **to be received by 1 May for the May/June examination series, and a date to be announced on the examination timetable for the winter examination series**. The name and address of the moderator will either be printed on the OPTEMS or supplied separately.

ABS – absent

W – withdrawn

* – sampled candidate

✔ – additional sampled candidates.

Where more than one teacher has assessed the work the teachers' initials should be given beside each candidate's name.

Centres are reminded that it is their responsibility to ensure that internal standardisation of the marking is carried out. The following **authentication** and internal standardisation statement should be written at the bottom of the printout and signed by the teacher responsible:

> '*I declare that the work of each candidate for whom marks are listed is*, *to the best of my knowledge*, *the candidate's own and that where several teaching groups are involved the marking has been internally standardised to ensure consistency across groups*.'
>
> Signed . . . . Date . . . .

Centres are advised to retain a copy of the annotated printout.

3 Coursework Record Sheets

A copy of the Coursework Record Sheet is provided on page 70 for centres to photocopy. The Coursework Record Sheet, to be completed for each candidate, provides details for the moderator of how each candidate's total mark is reached. It is the teacher's responsibility to ensure that:

all marks are recorded accurately and that the arithmetic is correct

the total mark is transferred correctly onto the OPTEMS or via EDI

any required authentication statement is signed by the candidate and/or teacher as appropriate.

Where a candidate's work is included in the sample the coursework record sheet should be attached to the work.

4 Sample of work for moderation

4.1 **Where the pre-printed OPTEMS is asterisked** indicating the candidates whose work is to be sampled, this work, together with the second copy of the OPTEMS, should be posted to reach the moderator by 1 May for candidates seeking certification in the summer series, and **a date to be announced on the examination timetable** for candidates seeking certification in the winter series. The name and address of the moderator will either be printed on the OPTEMS or supplied separately.

In addition, the centre must send the work of the candidate awarded the **highest** mark and the work of the candidate awarded the **lowest** mark, if these are not already included within the initial samples selected. The centre should indicate the additional samples by means of a tick (✔) in the left-hand column against the names of each of the candidates concerned.

For all sampled work the associated record sheet must be attached to each candidate's work.

If the pre-selected sample does NOT adequately represent ALL parts of the entire mark range for the centre, additional samples in the range(s) not covered should also be sent to the moderator. As above, additional samples should be indicated by means of a tick (✔).

For centres submitting marks by EDI the candidates in the sample selected on the OPTEMS should be marked with an asterisk (*) or a tick (✔), as appropriate, on the EDI printout. The annotated printout must be sent to the moderator with the sample of work.

4.3 **In all cases** please note that the moderator may request further samples of coursework, as required and the work of all candidates should be readily available in the event of such a request.

Internal standardisation

Centres are reminded that it is their responsibility to ensure that where more than one teacher has marked the work, internal standardisation has been carried out. This procedure ensures that the work of all candidates at the centre is marked to the same standards. The statement confirming this on the OPTEMS or the EDI printout must be signed.

Textbooks and resources

Core Textbooks

New editions of Heinemann's textbooks for GCSE Mathematics will be available for the start of courses leading to examinations covered by this specification. These revised editions will support the new Edexcel GCSE specification and will provide:

- one book covers each tier of entry
- coverage of the general topic areas of the specification
- new data handling project unit
- progression from key stage 3
- comprehensive banks of practice questions
- detailed, worked examples and examination questions
- key topic summaries
- examination style question papers

Revision Books

- a framework for structured revision
- key points to remember
- worked examination questions and examples
- revision exercises
- test yourself questions for students to gauge their progress
- links to chapters in core books

Practice Books

- additional exercises for homework and consolidation
- links to course text exercises
- concise and cost effective

Modular GCSE Revision and Practice Books

- one book for each of stages 1 and 2 of each tier
- consolidation and practice for each modular test in a single book
- key points to remember
- links to chapters in core books
- examination style practice modular test

Exambank

Heinemann will also be producing Exambank on CD ROM, which will enable centres to use past examination questions to:

- select past exam questions on chosen mathematical topics
- print immediately or save in Word files
- create tailored tests
- separate tests and answer sheets.

Details can be obtained from:

Heinemann Educational
Halley Court
Jordan Hill
Oxford OX2 8EJ

Tel: 01865 888080
Fax: 01865 314029
E-mail: orders@heinemann.co.uk

Support and training

Training

A programme of INSET courses covering various aspects of the specifications and assessment will be arranged by Edexcel each year on a regional basis. Full details may be obtained from:

INSET
Edexcel
Stewart House
32 Russell Square
London WC1B 5DN

Tel: 020 7393 4572
Fax: 020 7331 4046
E-mail: inset@edexcel.org.uk

Website

www.edexcel.org.uk

Please visit the Edexcel website, where further information about training and support for all qualifications, including this GCSE, can be found.

The website is regularly updated, and an increasing amount of support material and information will become available through it.

E–mail

Enquiries about this, or any other Edexcel mathematics qualification, can be made using the subject specific e–mail:

maths@edexcel.org.uk

Edexcel publications

Support materials and further copies of this specification can be obtained from:

Edexcel Publications
Adamsway
Mansfield
Notts NG18 4LN

Tel: 01623 467467
Fax: 01623 450481
E-mail: publications@linneydirect.com

The following support materials will be available from spring 2001 onwards:

- Specimen papers (1387 and 1388)
- Coursework guide – Using and Applying
- Coursework Guide – Data Handling
- Teachers' Guide – Content Exemplification (Linear)
- Teachers' Guide – Content Exemplification (Modular).

Regional offices and Customer Response Centre

Further advice and guidance is available through a national network of regional offices. For general enquiries and for details of your nearest office please call the Edexcel Customer Response Centre on 0870 240 9800.

Appendix 1: Assessment criteria for using and applying mathematics (tasks)

Mark	Making and monitoring decisions to solve problems	Communicating mathematically	Developing skills of mathematical reasoning
1	Candidates try different approaches and find ways of overcoming difficulties that arise when they are solving problems. They are beginning to organise their work and check results.	Candidates discuss their mathematical work and are beginning to explain their thinking. They use and interpret mathematical symbols and diagrams.	Candidates show that they understand a general statement by finding particular examples that match it.
2	Candidates are developing their own strategies for solving problems and are using these strategies both in working within mathematics and in applying mathematics to practical contexts.	Candidates present information and results in a clear and organised way, explaining the reasons for their presentation.	Candidates search for a pattern by trying out ideas of their own.
3	In order to carry through tasks and solve mathematical problems, candidates identify and obtain necessary information; they check their results, considering whether these are sensible.	Candidates show understanding of situations by describing them mathematically using symbols, words and diagrams.	Candidates make general statements of their own, based on evidence they have produced, and give an explanation of their reasoning.
4	Candidates carry through substantial tasks and solve quite complex problems by breaking them down into smaller, more manageable tasks.	Candidates interpret, discuss and synthesise information presented in a variety of mathematical forms. Their writing explains and informs their use of diagrams.	Candidates are beginning to give a mathematical justification for their generalisations; they test them by checking particular cases.

5	Starting from problems or contexts that have been presented to them, candidates introduce questions of their own, which generate fuller solutions.	Candidates examine critically and justify their choice of mathematical presentation, considering alternative approaches and explaining improvements they have made.	Candidates justify their generalisations or solutions, showing some insight into the mathematical structure of the situation being investigated. They appreciate the difference between mathematical explanation and experimental evidence.
6	Candidates develop and follow alternative approaches. They reflect on their own lines of enquiry when exploring mathematical tasks; in doing so they introduce and use a range of mathematical techniques.	Candidates convey mathematical meaning through consistent use of symbols.	Candidates examine generalisations or solutions reached in an activity, commenting constructively on the reasoning and logic employed, and make further progress in the activity as a result.
7	Candidates analyse alternative approaches to problems involving a number of features or variables. They give detailed reasons for following or rejecting particular lines of enquiry.	Candidates use mathematical language and symbols accurately in presenting a convincing reasoned argument.	Candidates' reports include mathematical justifications, explaining their solutions to problems involving a number of features or variables.
8	Candidates consider and evaluate a number of approaches to a substantial task. They explore extensively a context or area of mathematics which they are unfamiliar. They apply independently a range of appropriate mathematical techniques.	Candidates use mathematical language and symbols efficiently in presenting a concise reasoned argument.	Candidates provide a mathematically rigorous justification or proof of their solution to a complex problem, considering the conditions under which it remains valid.

Appendix 2: Elaboration of Ma1 assessment criteria

	Making and monitoring decisions to solve problems	Minimum Requirements	Notes
1/1	Candidates try different approaches and find ways of overcoming difficulties that arise when they are solving problems. They are beginning to organise their work and check results.	The candidate can, with help, understand a simple task and produce some information or results.	Simple task .e.g. matchstick row of squares – one or two random examples
1/2	Candidates are developing their own strategies for solving problems and are using these strategies both in working within mathematics and in applying mathematics to practical contexts.	The candidate interprets a simple task showing **some evidence** of their own planning and obtains a number of results, but no conclusion.	Planning may simply imply more calculations or results. Those may contain errors.
1/3	In order to carry through tasks and solve mathematical problems, candidates identify and obtain necessary information: they check their results, considering whether these are sensible.	The candidate obtains what is required to solve a simple task, finding and checking necessary information.	Checking: There should be no results that are obviously wrong e.g. 1, 2, 3, 5, 8, 25, 21 for Fibonacci. Checking is implied by correct results.
1/4	Candidates carry through substantial tasks and solve quite complex problems by breaking them down into smaller, more manageable tasks.	The candidate carries through a substantial task without additional direction, by breaking it down into smaller more manageable sub-tasks at least one of which is solved.	Substantial task: a task that needs to be subdivided into smaller tasks by the candidate in order to reach a solution.
1/5	Starting from problems or contexts that have been presented to them, candidates introduce questions of their own, which generate fuller solutions.	The candidate independently extends the task by changing one feature in order to give a fuller solution.	A feature is some aspect of the task such as a variable, constraint or condition
1/6	Candidates develop and follow alternative approaches. They reflect on their own lines of enquiry when exploring mathematical tasks; in doing so they introduce and use a range of mathematical techniques .	The candidate reflects on their line of enquiry and uses an additional relevant technique to extend the task further.	Reflects: Looks at and learns from their previous experience and moves the task on.
1/7	Candidates analyse alternative approaches to problems involving a number of features of variables. They give detailed reasons for following or rejecting particular lines of enquiry.	The candidate works on complex task(s) involving at least 3 features and gives reasons for following or rejecting lines of enquiry.	Complex task: Substantial involving at least 3 features requiring a range of techniques to reach a solution. The work must be at an appropriate level.
1/8	Candidates consider and evaluate a number of approaches to a substantial task. They explore extensively a context or area of mathematics with which they are unfamiliar. They apply independently a range of appropriate mathematical techniques.	The candidate applies, independently, appropriate mathematical techniques extensively to solve a complex problem.	The mathematics should be from the National Curriculum 'further material' or beyond.

In strand 1: a maximum of 3 marks should be awarded in relation to "simple" tasks; a maximum of 6 marks should be awarded to "substantial tasks; a maximum of 8 marks should be awarded in relation to "complex" tasks.

To qualify for a mark of 4, 6, 8 on any strand the content of the task must meet, or go beyond, the relevant aspects of the grade descriptors for grades F, C and A respectively.

	Communicating Mathematically	**Minimum Requirements**	**Notes**
2/1	Candidates discuss their mathematical work and are beginning to explain their thinking. They use and interpret mathematical symbols and diagrams.	The candidate shows some evidence of their thinking	This may be oral, (supported by teacher annotation), or written and could take the form of random calculations or drawings etc.
2/2	Candidates present information and results in a clear and organised way, explaining the reasons for their presentation.	The candidates present some information or results in a clear or organised way.	This could include listing and/or diagrams
2/3	Candidates show understanding of situations by describing them mathematically using symbols, words and diagrams.	The candidate shows some understanding of the task by describing a feature of the task mathematically by using words and symbols **or** symbols and diagrams.	Words can be headings, statements or connectives. It could be shown by a list with "lettered" headings.
2/4	Candidates interpret, discuss and synthesise information presented in a variety of mathematical forms. Candidates' writing explains and informs their use of diagrams.	The candidate brings together more than one form of mathematical presentation with a **linking** commentary.	This is not a series of displays or diagrams included for no purpose. The commentary must allow the reader to understand what the candidate has done.
2/5	Candidates examine critically and justify their choice of mathematical presentation, considering alternative approaches and explaining improvements they have made.	The candidate gives some explanation for their choice of presentation may be symbolic or diagrammatic.	The introduction of a simple algebraic formula does not justify 6 marks in this strand but, as best fit, could achieve 5 marks. Key words such as "because", "therefore", "hence", "since", … could be used when justifying improvements.
2/6	Candidates convey mathematical meaning through consistent use of symbols.	The candidate **conveys** mathematical meaning through the sustained **use of symbolism*** at the appropriate level	Variables need to be defined and symbols must be correctly used in a number of cases. Some minor errors or omissions may occur without penalty.
2/7	Candidates use mathematical language and symbols accurately in presenting a convincing reasoned argument.	The candidate presents a convincing reasoned argument through the use of mathematical language and symbolism which is generally accurate.	There should be increased emphasis on accuracy. Incorrect algebra cannot lead to a convincing argument.
2/8	Candidates use mathematical language and symbols efficiently in presenting a concise reasoned argument.	The candidate produces an elegant argument.	

To qualify for a mark of 4, 6, 8 on any strand the content of the task must meet, or go beyond, the relevant aspects of the grade descriptors for grades F, C and A respectively.

* Symbolism might include for example: 1 Algebra
2 Trigonometry

	Developing the skills of mathematical reasoning	**Minimum Requirements**	**Notes**
3/1	Candidates show that they understand a general statement by finding particular examples that match it.	The candidate produces a simple example that shows an understanding of the task	
3/2	Candidates search for a pattern by trying out ideas of their own.	The candidate gathers sufficient data from which a simple observation may be made.	In most situations this would involve at least 3 results.
3/3	Candidates make general statements of their own, based on evidence they have produced, and give an explanation of their reasoning.	The candidate makes a general statement based on **their** results.	Their result need not be correct for the task but should be consistent with their data. A general statement might be as simple as “goes up in 2’s” or “all odd numbers”.
3/4	Candidates are beginning to give a mathematical justification for their generalisation; they test them by checking particular cases.	The candidate tests their generalisation by checking a further case.	A test is a prediction with a confirmation from the mathematical situation of the problem using new data.
3/5	Candidates justify their generalisation or solutions, showing some insight into the mathematical structure of the situation being investigated. They appreciate the difference between mathematical explanation and experimental evidence.	The candidate produces a sensible argument stating **why** the results occur by relating these results to he mathematical situation e.g. physical, geometrical or graphic	For example explaining why coefficients and constants in a generalisation occur, not simply from difference tables.
3/6	Candidates justify their generalisation or solutions reached in an activity, commenting constructively on the reasoning and logic employed, and make further progress in the activity as a result.	The candidate uses reasoning and logic to make further progress in the activity.	
3/7	Candidates’ reports include mathematical justifications, explaining their solutions to problems involving a number of features or variables.	The candidate gives a general result or conclusion with justification for parts of the overall solution, co-ordinating at least 3 features.	This mark cannot be awarded without the award of 7 or 8 marks in strand 1.
3/8	Candidates provide a mathematically rigorous justification or proof of their solution to a complex problem, considering the conditions under which it remains valid.		

To qualify for a mark of 4, 6, 8 on any strand the content of the task must meet, or go beyond, the relevant aspects of the grade descriptors for grades F, C and A respectively.

Appendix 3: Task form

GCSE Mathematics Specifications A and B (1387/8)

Coursework Record Form

Candidate Name ___________________________ Candidate No. ____________________

Centre Name _______________________________ Centre No: ________________________

Task ____________________________________ Project: ___________________________

Date _____________ Date __________________

Tier of Entry___________________

Overall Total Mark (out of 48) __________

Task 1		Task 2 (optional)		Project	
Strand	Mark	Strand	Mark	Area	Mark
1		1		1	
2		2		2	
3		3		3	

Help given over and above normal classroom practice

Date	Nature of Help

Candidate's oral contribution

Candidate's practical work

DECLARATION TO BE SIGNED BY THE TEACHER-EXAMINER RESPONSIBLE FOR COMPLETING THE TASK FORM

I declare that the task and project of the candidate in respect of the marks on this form have been kept under regular supervision and that, to the best of my knowledge, no assistance has been given apart from any which is acceptable under the scheme of assessment and has been identified and recorded.

Signed ……………………………………………………….. **Date** ……………………………

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E-mail: publications@linneydirect.com

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